The Multiple Absorption Coefficient Zonal Method (MACZM), an Efficient Computational Approach Radiative Heat Transfer in Multi-Dimensional Inhomogeneous Non-gray Media

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ABSTRACT

The formulation of a multiple absorption coefficient zonal method (MACZM) is presented. The concept of generic exchange factors (GEF) is introduced. Utilizing the GEF concept, MACZM is shown to be effective in simulating accurately the physics of radiative exchange in multi-dimensional inhomogeneous non-gray media. The method can be directly applied to a fine-grid finite-difference or finite-element computation. It is thus suitable for direction implementation in an existing CFD code for analysis of radiative heat transfer in practical engineering systems.

The feasibility of the method is demonstrated by calculating the radiative exchange between a high temperature (~3000 K) molten nuclear fuel (UO₂) and water (with a large variation in absorption coefficient from the visible to the infrared) in a highly 3-D and inhomogeneous environment simulating the premixing phase of a steam explosion.

NOMENCLATURE

\( a \) = absorption coefficient
\( A \) = area element
\( dA \) = differential area element
\( dV \) = differential volume element
\( D \) = length scale (grid size) of the discretization
\[ F_{zzz} = \text{dimensionless volume-volume exchange factor, Eq. (11a)} \]
\[ F_{gzz} = \text{dimensionless volume-volume exchange factor, Eq. (11b)} \]
\[ F_{gz} = \text{dimensionless volume-surface exchange factor, Eq. (14a)} \]
\[ F_{gxx} = \text{dimensionless volume-surface exchange factor, Eq. (14b)} \]
\[ g_1g_2 = \text{volume-volume exchange factor, Eq. (1)} \]
\[ g_1s_2 = \text{volume-surface exchange factor, Eq. (5)} \]
\[ L_c = \text{characteristic lengths between two elements along the selected optical path} \]
\[ L_{mb} = \text{mean beam length between two volume (area) elements, Eq. (16)} \]
\[ \bar{n} = \text{unit normal vector} \]
\[ n_x, n_y, n_z = \text{dimensionless distance coordinate, Eq. (12)} \]
\[ r = \text{distance between volume elements, Eq. (3)} \]
\[ s = \text{distance, Eq. (4)} \]
\[ s_1s_2 = \text{surface-surface exchange factor, Eq. (6)} \]
\[ V = \text{volume element} \]
\[ Q = \text{heat transfer} \]
\[ T = \text{temperature} \]
\[ x = \text{coordinate} \]
\[ y = \text{coordinate} \]
\[ z = \text{coordinate} \]
\[ \sigma = \text{Stefan Boltzmann constant} \]
\[ \tau = \text{optical thickness, Eq. (3)} \]

**Subscripts**

1,2 = label of volume (area) element
INTRODUCTION

The ability to assess the effect of radiation heat transfer in multi-dimensional inhomogeneous media is important in many engineering applications such as the analysis of practical combustion systems and the mixing of high temperature nuclear fuel (UO₂) with water in the safety consideration of nuclear reactors. The lack of a computationally efficient and accurate approach, however, has been a major difficulty limiting engineers and designers from addressing many of these important engineering issues accounting for the effect of thermal radiation.

For example, in the analysis of steam explosion in a reactor safety consideration, it is important for account for the radiative exchange between hot molten material (e.g. UO₂) and water. The absorption coefficient for water is plotted together with the blackbody emissive power at 3052 K (the expected temperature of molten UO₂ in a nuclear accident scenario) in Figure 1. The radiative exchange between water and UO₂ must account for the highly nongray and rapidly increasing (by more than two order of magnitude) characteristic of the absorption coefficient of water. The multi-dimensional and inhomogeneous aspect of the “premixing” process are illustrated by Figure 2. In this particular physical scenario, molten UO₂ is released from the top into a cylindrical vessel with an annular overflow chamber as shown in the figure. Even with highly subcooled water (say, 20 C at 1 atm), voiding occurs quickly leading to a complex two phase mixture surrounding the hot molten UO₂. The radiative heat transfer between the hot molten UO₂ and the surrounding water is a key mechanism controlling the boiling process. The boiling process, on the other hand, depends on the radiative heat transfer and thus the amount of liquid water surrounding the hot molten material. An accurate assessment of this interaction is key to the understanding of this “premixing” process and ultimately to the resolution of the critical issue of steam explosion in the consideration of reactor safety.

Over the years, the zonal method has been shown to be an effective approach to account for the multi-dimensional aspect of radiative heat transfer in homogeneous and isothermal media [1]. This method was later extended for application to inhomogeneous and non-isothermal media with the concept of “generic” exchange factors (GEF) [2]. The underlying principle of the extended zonal method is that if a set of generic exchange factors with standard geometry is tabulated, the radiative exchange between an emitting element and an absorbing element of arbitrary geometry can be generated by superposition. The inhomogeneous nature can be
accounted for by using the appropriate average absorption coefficient in the evaluation of the
generic exchange factor. As grid size decreases, it is expected that the accuracy of the
superposition will increase. The error of using a single average absorption to account for the
absorption characteristics of the intervening medium will also decrease.

While the extended zonal method was effective in accounting for the effect of an
inhomogeneous medium in some problems [2], the accuracy of the approach for general
application is limited. Specifically, by using a set of GEF which depends on only a single
average absorption coefficient, the method do not simulate correctly the physics of radiative
exchange between two volume elements which depends generally on at least three characteristic
absorption coefficients (namely, the absorption coefficient of the emitting element, the
absorption coefficient of the absorbing element and the average absorption coefficient of the
intervening medium). A reduction in grid size cannot address this fundamental limitation.

In addition, the concept of a single average absorption coefficient for the intervening
medium is also insufficient, particularly in an environment where there is a large discontinuity of
the absorption coefficient. For example, consider the radiative exchange between a radiating
cubical water element \( V_1 \) and an absorbing cubical water element \( V_2 \) as shown in Figure 3. The
absorbing element \( V_2 \) is an element at the liquid/vapor phase boundary. It is adjacent to another
element of liquid water on one side while surrounded by a medium which is effectively optically
transparent. As shown in the same figure, there are two possible optical paths, indicated as \( S_1 \)
and \( S_2 \), over which the average absorption coefficient can be evaluated. For the physical
dimensions as shown in the figure, the average absorption coefficient evaluated along the optical
path \( S_2 \) increases from 6.38 \( \text{1/cm} \) to 306 \( \text{1/cm} \) as the wavelength increases from 0.95 \( \mu \text{m} \) to 3.27
\( \mu \text{m} \) while the average absorption coefficient evaluated along the optical path \( S_1 \) remains
effectively at zero (ignoring the very small absorption by water vapor). It would be difficult to
evaluate the radiative exchange between these two elements accurately using a single exchange
factor based on a single average absorption coefficient for the intervening medium. This large
discrepancy in the average absorption coefficient of the two optical paths remains even in the
limit of small grid size.

The objective of the present work is to present the mathematical formulation of a
multiple absorption coefficient zonal method (MACZM) which is mathematically consistent
with the physics of radiative absorption. The method will be shown to be efficient and accurate
in the simulation of radiative heat transfer in inhomogeneous media. A set of “three absorption coefficient” volume-volume exchange factors and “two absorption coefficient” volume-surface exchange factors are tabulated for rectangular elements. The generic exchange factor (GEF) concept is expanded to a two-component formulation to account for the possible large variation of absorption coefficient in regions surrounding the absorbing or emitting elements. Based these two-component generic exchange factors, the multi-dimensional and non-gray effect in any discretized domain can be evaluated accurately and efficiently by superposition. The accuracy of the superposition procedure is demonstrated by comparison with results generated by direct numerical integration. The characteristics of radiative exchange in a highly multi-dimensional, inhomogeneous and non-gray media such as those existed in the premixing phase of a steam explosion (as shown in Figure 2) are presented to illustrate the feasibility of the approach.

MATHEMATICAL FORMULATION

General Formulation

The basis of the zonal method [1] is the concept of exchange factor. Mathematically, the exchange factor between two discrete volumes, \( V_1 \) and \( V_2 \), in a radiating environment is

\[
g_{12} = \int \int_{V_1, V_2} \frac{a_1 a_2 e^{-\tau} dV_1 dV_2}{\pi r^2}
\]  

(1)

where

\[
r = \left[ (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2 \right]^{1/2}
\]  

(2)

\( \tau \) is the optical thickness between the two differential volume elements, \( dV_1 \) and \( dV_2 \), given by
\[ \tau = \int_{r_1}^{r_2} a(s) \, ds \quad (3) \]

with \( a \) being the absorption coefficient and

\[ s = |\vec{r} - \vec{r}_1| \quad (4) \]

The integration in Eq. (3) is performed along a straight line of sight from \( r_1 \) to \( r_2 \).

In a similar manner, the exchange factor between a volume element \( V \) and a surface element \( A \) and that between two area elements \( A_1 \) and \( A_2 \) are given, respectively, by

\[ g_{12} = \int \int_{V_1A_2} a e^{-\tau} |\vec{n}_2 \cdot \vec{r}| dV_1 dA_2 \quad (5) \]

\[ s_{12} = \int \int_{A_1A_2} e^{-\tau} |\vec{n}_1 \cdot \vec{r}| |\vec{n}_2 \cdot \vec{r}| dA_1 dA_2 \quad (6) \]

where \( \vec{n}_1 \) and \( \vec{n}_2 \) are unit normal vectors of area elements \( dA_1 \) and \( dA_2 \).

It should be noted that Eqs. (1), (5) and (6) are applicable for general inhomogeneous non-scattering media in which the absorption coefficient is a function of position. Physically, the exchange factor can be interpreted as the fraction of energy radiated from one volume (or area) and absorbed by a second volume (or area). Specifically, for a volume \( V_1 \) with uniform temperature \( T_1 \), the absorption by a second volume \( V_2 \) of radiation emitted by \( V_1 \) is given by
and the absorption by a black surface $A_2$ of radiation emitted by $V_1$ is given by

$$Q_{r_1 \rightarrow r_2} = \sigma T_1^4 g_1 g_2$$  \hspace{1cm} (7)$$

Similarly, for a black surface $A_i$ with uniform temperature $T_i$, the absorption by a volume $V_2$ of radiation emitted by $A_i$ is given by

$$Q_{A_i \rightarrow r_2} = \sigma T_i^4 g_i g_2$$  \hspace{1cm} (9a)$$

where, by reciprocity,

$$g_i g_2 = g_2 g_i$$  \hspace{1cm} (9b)$$

Finally, the absorption by a black surface $A_2$ of radiation emitted by $A_i$ is given by

$$Q_{A_1 \rightarrow A_2} = \sigma T_1^4 s_i s_2$$  \hspace{1cm} (10)$$

The Discretization

The evaluation of Eqs. (7) to (10) in a general transient calculation in which the spatial distribution of the absorption coefficient is changing (for example, due to the change in the
spatial distribution of hot materials and void fraction during the “premixing” process as shown in Figure 2) is too time consuming even with fast computers. Anticipating that all calculations will be generally done in a discretized computational domain, it is useful to develop a set of “generic” exchange factors (GEF) which will be applicable for all calculations.

Specifically, consider the geometry as shown in Figure 4. Assuming that the absorption coefficient within the two discrete volumes \( a_1 \) and \( a_2 \) are constant, MACZM introduces two partial exchange factors, \((g_{zz}, g_{xz})\) to characterize the radiative exchange between the two volumes. The partial exchange factor \((g_{zz})\) represents the radiative exchange between the two volume consisting only of those energy rays which pass through the top surface of \( V_1 \) \((z = z_1 + D)\) and the bottom surface of \( V_2 \) \((z = z_1 + n_y D)\). The factor \((g_{xz})\), on the other hand, represents the radiative exchange between the two volume consisting only of those energy rays which pass through the “x-direction” side surface of \( V_1 \) \((x = x_1 + D)\) and the bottom surface of \( V_2 \) \((z = z_1 + n_y D)\). Assuming that the absorption coefficient of the intervening medium is constant (but different for the two partial exchange factors), the two partial exchange factors can be expressed in the following dimensionless form

\[
\frac{(g_{zz})}{D^2} = F_{zz} \left( a_1 D, a_2 D, a_{m,zz}, D, n_x, n_y, n_z \right)
\]

\[
\frac{(g_{xz})}{D^2} = F_{xz} \left( a_1 D, a_2 D, a_{m,xz}, D, n_x, n_y, n_z \right)
\]

with

\[
n_x = \frac{x_2 - x_1}{D}, \quad n_y = \frac{y_2 - y_1}{D}, \quad n_z = \frac{z_2 - z_1}{D}
\]

The two functions \( F_{zz} \left( a_1 D, a_2 D, a_{m,zz}, D, n_x, n_y, n_z \right) \) and \( F_{xz} \left( a_1 D, a_2 D, a_{m,xz}, D, n_x, n_y, n_z \right) \) are dimensionless functions of the three optical thicknesses \( (a_1 D, a_2 D, a_{m,zz} D \text{ or } a_{m,xz} D) \) and the
dimensionless separation between the two volume elements \((n_x, n_y, n_z)\). For a rectangular
discretization with constant grid size \((dx = dy = dz = D)\), these dimensionless distances only take
on discretized value, i.e. \(n_x, n_y, n_z = 0, 1, 2 \cdots\). The two dimensionless function tabulated at
different optical thicknesses \((a_1D, a_2D, a_{m,zz}D \text{ or } a_{m,xz}D)\) and discretized values of \((n_x, n_y, n_z)\)
constitutes two sets of “generic” exchange factor (GEF) which will be applicable for all
calculations with uniform grid size. The intervening absorption coefficient \(a_m\) is the average of
the absorption coefficient taken along a line of sight directed from the center of the top area
element of \(V_1\) \((z = z_1 + D)\) to the center of the bottom surface of \(V_2\) \((z = z_1 + n_zD)\).
Similarly,

Mathematically, the exchange factor between the two cubical volumes can be generated
from Eqs. (11a) and (11b) by superposition as

\[
\frac{g_1g_2}{D^2} = F_{gxxz} (a_1D, a_2D, a_{m,xx}D, n_x, n_y, n_z) \\
+ F_{gzxz} (a_1D, a_2D, a_{m,zz}D, n_x, n_y, n_z) \\
+ F_{gxyz} (a_1D, a_2D, a_{m,yz}D, n_x, n_y, n_z) \\
+ F_{gzyz} (a_1D, a_2D, a_{m,yx}D, n_x, n_y, n_z) \\
+ F_{gxxz} (a_1D, a_2D, a_{m,zz}D, n_x, n_y, n_z) \\
+ F_{gxyz} (a_1D, a_2D, a_{m,yy}D, n_x, n_y, n_z) \\
+ F_{gzyz} (a_1D, a_2D, a_{m,xy}D, n_x, n_y, n_z) \\
+ F_{gxxz} (a_1D, a_2D, a_{m,zz}D, n_x, n_y, n_z) \\
+ F_{gxyz} (a_1D, a_2D, a_{m,yx}D, n_x, n_y, n_z) \\
+ F_{gzyz} (a_1D, a_2D, a_{m,yx}D, n_x, n_y, n_z) \\
+ F_{gzyz} (a_1D, a_2D, a_{m,xx}D, n_x, n_y, n_z)
\] (13)

Eq. (13), together with the tabulated values of the two GEF’s, \(F_{gxxz} (a_1D, a_2D, a_{m,zz}D, n_x, n_y, n_z)\)
and \(F_{gzxz} (a_1D, a_2D, a_{m,xz}D, n_x, n_y, n_z)\), contain all the essential physics needed to characterize the
radiative exchange between the two elements. It accounts for the absorption characteristics of
the absorbing and emitting element \((a_1D, a_2D)\). By using different average absorption
coefficients \( (a_{m,pq} D, \ p, q = x, y, z) \) for the intervening medium, it accounts for not only the absorption characteristics of the intervening medium, but also and the variation of absorption characteristics in the neighborhood of the absorbing and emitting elements (such as the situation as shown in Figure 3).

The exchange factor \( g_{1s_2} \) can be similarly expressed in a dimensionless form. Using the geometry as shown in Figure 5, two partial exchange factors, \( (g_{1s_2})_z \) and \( (g_{1s_2})_x \), are introduced. Physically, the partial exchange factor \( (g_{1s_2})_z \) represents the radiative exchange between \( V_1 \) and \( A_2 \) consisting only of those energy rays which pass through the top surface of \( V_1 \) \((z = z_1 + D)\). The factor \( (g_{1s_2})_x \), on the other hand, represents the radiative exchange between \( V_1 \) and \( A_2 \) consisting only of those energy rays which pass through the “x-direction” side surface of \( V_1 \) \((x = x_1 + D)\). Assuming that the absorption coefficient of the intervening medium is constant (but different for the two partial exchange factors), the two partial exchange factors can be expressed in the following dimensionless form

\[
\frac{(g_{1s_2})_z}{D^2} = F_{g_{z}} \left( a_1 D, a_{m,z} D, n_x, n_y, n_z \right) \tag{14a}
\]

\[
\frac{(g_{1s_2})_x}{D^2} = F_{g_{x}} \left( a_1 D, a_{m,x} D, n_x, n_y, n_z \right) \tag{14b}
\]

Note that in Figure 5, the area \( A_2 \) is assumed to be parallel to the x-y plane. For general application, there is no loss of generality since a discretized area is always parallel to one of the face of the discretized volume in a rectangular coordinate system with equal grid size. The two average absorption coefficients are taken along the two line of sights directed toward the center of the receiving plane, from the top area element \((z = z_1 + D)\) and x-direction side area element \((x = x_1 + D)\) respectively. Similar to Eq. (13), the exchange factor between between \( V_1 \) and \( A_2 \) can be generated by superposition as
The exchange factor $s_1s_2$ is a function of only one average absorption coefficient for the intervening medium ($a_m$). Its formulation and mathematical behavior have already been presented and discussed in the earlier work [2] and will not be repeated here.

**The “Generic” Exchange Factor (GEF) and its Properties**

Numerical data for the “generic” exchange factors are generated in this section to illustrate the mathematical behavior of the exchange factor. For a practical calculation, these factors can be tabulated as a “look-up” table based on which the radiative exchange can be computed accurately and efficiently by superposition.

Since GEF are functions only of optical thicknesses and geometric orientation, the accuracy of the superposition procedure is generally insensitive to the physical dimension $D$ (i.e. the grid size). As an illustration, the radiative exchange between a volume element and area element as shown in Figure 6 is considered. The superposition solutions are generated by subdividing the volume and area into cubical volume and area elements with dimension $\Delta$. A comparison between the superposition solution and that generated by direct numerical integration is shown in Table 1. For the two volume elements as shown in Figure 7, a similar comparison is shown in Table 2. In both cases, the accuracy of the superposition results appears to be somewhat insensitive to the dimension $\Delta$, the slight discrepancy can be attributed to the slight error in the interpolation of the “look-up” table over discrete optical thicknesses. The numerical data presented in the two tables, for example, are generated with a set of GEF tabulated for $a_1D, a_2D, a_mD = 0, 0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0$. The discrepancy will decrease as the number of data points in the “look-up” table increases. For practical application, the grid size is important only in terms of how well the rectangular discretization simulates the actual geometry. When the geometry is simulated accurately, the accuracy of MACZM depends only on the number of discrete data points used in the GEF table.
One important concept which has been used frequently by the practical engineering community to account for the multi-dimensional effect of radiation is the concept of “mean beam length”. But until now, this concept has been applied generally to homogeneous medium and the verification of the accuracy of the approach for multi-dimensional, inhomogeneous and non-gray applications is quite limited [3]. In the current formulation of the MACZM and the associated GEF, numerical data show that the concept of mean beam length can be readily applied to provide a simplified mathematical characterization of the effect of the intervening medium. Specifically, a concept of mean beam length, $L_{mb}$, can be introduced by

$$F(a_m D, n_x, n_y, n_z) = F(0, n_x, n_y, n_z)e^{-a_m L_{mb}}$$  \hspace{1cm} (16)$$

where the function $F(a_m D, n_x, n_y, n_z)$ represents any one of the four GEF’s $(F_{gzz}(a_1 D, a_2 D, a_{m,zz} D, n_x, n_y, n_z), F_{gzz}(a_1 D, a_2 D, a_{m,zz} D, n_x, n_y, n_z), F_{gzz}(a_1 D, a_{m,z} D, n_x, n_y, n_z)$ and $F_{gzz}(a_1 D, a_{m,z} D, n_x, n_y, n_z)$) and $a_m$ is the corresponding average absorption coefficient $(a_{m,zz}, a_{m,zz}, a_{m,z}, a_{m,z})$. Mean beam lengths for the four GEF’s for some typical geometry are tabulated and shown in Figure 8. The numerical data show that the mean beam length for the four GEF’s are generally functions only of geometry and is remarkably independent of all of optical thicknesses.

Physically, the mean beam length is expected to be approximately the characteristic distance between the emitting element and the absorbing element. To illustrate the dependency, the mean beam length can be written as

$$L_{mb} = C L_c(n_x, n_y, n_z)$$  \hspace{1cm} (17)$$

where $L_c(n_x, n_y, n_z)$ is taken to be the length of the line of sight over which the average absorption coefficient is evaluated for the four GEF’s. Numerical data for C, based on the average value of the mean beam length (taken over the different optical thicknesses), are generated for different geometry ($n_x, n_y, n_z = 0, 5$ for $F_{gzz}$ and $F_{gzz}$, $n_x, n_y = 0, 5$ and $n_z = 1, 10$
for \( F_{gsx} \) and \( F_{gsz} \). These data are plotted collectively against a single variable \( (n_z) \) in Figure 9.

It is interesting to note that \( C \) is close to unity and is approximately constant except for configurations in which the emitting and absorbing elements are close to each other. For combustion gases, this mathematical behavior of \( C \) can be utilized to develop band correlation to characterize the absorption of the intervening gas. This effort is currently under consideration and will be presented in future publications.

**APPLICATION**

MACZM is applied to analyze the effect of radiation on the mixing of hot molten fuel with water. For simplicity, the radiative absorption of steam is neglected in the calculation. The detailed analysis and results will be presented in future publications. In the present work, the predicted radiative heat transfer distribution is presented to illustrate the effectiveness of MACZM.

Because of the large variation of the absorption coefficient of water over the wavelength of interest as shown in Figure 1, a three-band approach is used to capture the difference in radiative energy distribution in the different wavelength region. The step wise approximation used for the absorption coefficient of water is shown in Figure 10. The absorption coefficients of the three bands correspond to the absorption coefficient of three characteristic wavelengths 0.4915 \( \mu \)m, 0.9495 \( \mu \)m and 3.277 \( \mu \)m respectively. The middle wavelength (0.9495 \( \mu \)m) is the wavelength at which the blackbody emissive power at the molten fuel temperature (3052 K) is a maximum. The fractions of energy radiated by the molten fuel (at 3052 K) for the three bands are 0.125, 0.647 and 0.228 respectively. Using a grid size of 10 cm (with the inner vessel diameter of 70 cm), the rate of energy absorption by water predicted for three different times during the premixing transient are shown in Figures 11a, 11b and 11c. It can be readily observed that the distribution of water energy absorption varies significantly among the three bands. In the first band at which water is optically transparent, the radiation penetrates a significant distance away from the radiating molten fuel. This accounts for the “red hot” visual appearance commonly observed in the interaction of high temperature molten fuel and water. The first band, however, accounts only for 12.5% of the total energy radiated from the fuel. For the remaining energy, the water absorption coefficient is high and the water absorption is highly localized in the region surrounding the fuel. The localized absorption appears to dominate the boiling
process as the second and third band account for more than 80% of the radiative emission. MACZM captures both the transient and spatial distribution of the radiative absorption distribution accurately and efficiently.

Because of the large variation of the water absorption coefficient over wavelength and the large values of the water absorption coefficient in the long wavelength region, a larger number of band and smaller grid size are needed to simulate accurately the effect of radiation on the premixing process. This effort is currently underway and results will be presented in future publications.

CONCLUSION

The formulation of a multiple absorption coefficient zonal method (MACZM) is presented. Four “generic” exchange factors (GEF) are shown to be accurate and effective in simulating the radiative exchange. Numerical values these GEF’s are tabulated and their mathematical behavior is described. The concept of mean beam length is shown to be effective in separating the effect of the intervening absorption coefficient on the radiative exchange.

MACZM is shown to be effective in capturing the physics of radiative heat transfer in a multi-dimensional inhomogeneous three phase mixture (molten fuel, liquid and vapor) generated in the premixing phase of a steam explosion.

REFERENCES

Figure 1: The absorption coefficient of water and the blackbody emissive power at 3052 K.
Figure 2: The distribution of molten UO2 (left, with the black dot representing the “fuel” as lagrangian particles) and the void fraction distribution of water (right) during a premixing process.
Figure 3: Example geometry highlighting the difference in “average absorption coefficient” for different optical path.
Figure 4: Geometry and coordinate system used in the definition of the $g_1g_2$ GEF.
Figure 5: Geometry and coordinate system used in the definition of the \( g_{fs} \) GEF.
Figure 6: Geometry and coordinate system used in the illustration of the accuracy of the superposition procedure for the evaluation of the exchange factor $g, s_2$. 
Figure 7: Geometry and coordinate system used in the illustration of the accuracy of the superposition procedure for the evaluation of the exchange factor $g_1 g_2$. 
Figure 8: Mean beam lengths of the four GEF for some selected geometry.
Figure 9: Values of \( \frac{L_{mb}}{L_c} \) of the four GEF for different values of \( n_x, n_y, n_z \).
Figure 10: The 3-band approximation of the water absorption coefficient used in the premixing calculation.
Figure 11a: The distribution of radiative absorption by water in the three absorption band (the right three figures) at 0.6 s after the initial pour predicted by the premixing calculation. The first figure on the left represents the distribution of the molten fuel (the black dots are the lagrangian particles representing fuel) and the second figure represents the void fraction distribution.
Figure 11b: The distribution of radiative absorption by water in the three absorption band (the right three figures) at 0.8 s after the initial pour predicted by the premixing calculation. The first figure on the left represents the distribution of the molten fuel (the black dots are the lagrangian particles representing fuel) and the second figure represents the void fraction distribution.
Figure 11c: The distribution of radiative absorption by water in the three absorption band (the right three figures) at 1.0 s after the initial pour predicted by the premixing calculation. The first figure on the left represents the distribution of the molten fuel (the black dots are the lagrangian particles representing fuel) and the second figure represents the void fraction distribution.
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<td>0.259e-1, 0.172e-1, 0.118e-1, 0.423e-2</td>
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Table 1: Comparison between the exchange factor generated by direct numerical integration and those generated by superposition of GEF for the geometry of Figure 6. ($\Delta$ is the length scale of the element used in the GEF superposition).
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<th>m</th>
<th>a_{1D}</th>
<th>a_{nD}</th>
<th>a_{2D}</th>
<th>Δ/D</th>
<th>g_{12} (a_{1D}, a_{2D}, a_{nD}, 0, 0, m)</th>
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<td>0.193e-1, 0.147, 0.390, 0.133e+1</td>
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<td>0.198e-1, 0.150, 0.391, 0.133e+1</td>
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<td>0.257e-1, 0.705e-1, 0.109, 0.181</td>
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<td>0.255e-1, 0.708e-1, 0.108, 0.181</td>
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<td>0.252e-1, 0.700e-1, 0.108, 0.180</td>
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<td>0.761e-3, 0.210e-2, 0.323e-2, 0.535e-2</td>
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<td>0.760e-3, 0.208e-2, 0.319e-2, 0.528e-2</td>
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<td>0.685e-3, 0.187e-2, 0.288e-2, 0.476e-2</td>
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<td>0.450e-3, 0.124e-2, 0.191e-2, 0.316e-2</td>
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<td>0.536e-2, 0.146e-1, 0.225e-1, 0.373e-1</td>
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<td>0.482e-2, 0.133e-1, 0.204e-1, 0.339e-1</td>
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<td>0.483e-2, 0.132e-1, 0.203e-1, 0.334e-1</td>
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<td>0.316e-2, 0.872e-2, 0.134e-1, 0.222e-1</td>
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<td>0.321e-2, 0.875e-2, 0.135e-1, 0.222e-1</td>
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<td>numerical</td>
<td>0.187e-2, 0.515e-2, 0.791e-2, 0.131e-1</td>
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<td>0.191e-2, 0.521e-2, 0.800e-2, 0.131e-1</td>
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</table>

Table 2: Comparison between the exchange factor generated by direct numerical integration and those generated by superposition of GEF for the geometry of Figure 7. (Δ is the length scale of the element used in the GEF superposition).