1 Introduction

There is a growing demand for effective and efficient signal filters, due to their presence in a wide variety of wireless communications devices, including cellular phones. Most previous work on such filters has focused on conventional electrical filters [1] and their mechanical counterparts [2]. More recently, the introduction of microelectromechanical (MEM) devices into mainstream engineering has opened a promising new path of study regarding the development of filters [3–7]. This promise is founded upon the distinct advantages MEM filters hold over more conventional designs. Specifically, MEM filters are inherently smaller, consume less power to operate, and have the potential to operate with significantly higher quality (Q) factors (in the case of bandpass filters, resonant frequency to bandwidth ratios) [8]. In addition, MEM filters are easy to integrate with electrical circuitry (sometimes on a single chip), are highly tunable, and, most importantly for this work, MEM oscillators can be designed to exhibit parametric resonance [9]. Virtually all MEM bandpass filters proposed to date have utilized features of direct linear resonance to achieve the desired filtering [5–7]. In contrast, here we describe a means of using parametric resonance in MEMS as the basis for filtering.

The paper is arranged as follows. In Sec. 2 we review some basics regarding filter performance, and in Sec. 3 some of the general features of parametric resonance and their relevance to filtering are briefly described. In Sec. 4 the behavior of a parametrically excited MEM oscillator is analyzed, and in Sec. 5 we describe a means of manipulating the oscillator’s instability zone in a manner that is beneficial for filter design. The required nonlinear tuning of the oscillators is outlined in Sec. 6, and the logic for creating a bandpass filter using a pair of MEM oscillators is described in Sec. 7. The paper closes in Sec. 8 with a brief discussion of the results and some outstanding issues related to this line of work.

2 Bandpass Filter Basics

The focus in this paper is bandpass filters, which pass signals with frequency components inside a specified passband, attenuating those outside the passband. Figure 1 highlights some of the key features considered when assessing the performance of such filters in the context of a frequency response transmission function. The following characteristics are of particular interest:

- The center passband frequency—the nominal operating frequency of the filter.
- The bandwidth—the range of frequencies that will pass through the filter with minimal loss in signal strength.
- The stopband rejection—the amount by which the signal is attenuated outside of the passband.
- The insertion loss—the reduction in signal amplitude as the signal passes through the filter.
- The sharpness of the rolloff—the width of the frequency range between the edges of the passband and the stopband.
- The flatness of the passband response—the degree to which ripples are present in the filter’s passband frequency response.

An ideal filter would pass the entire signal, unaffected, in the passband, and completely reject signals outside the passband. In virtually all filter designs, this is approximated using the resonance features of a set of weakly coupled linear oscillators with very low damping, implemented using electrical circuits or mechanical oscillators, which often use surface acoustic waves (so-called SAW filters) [10–13], bulk acoustic waves (FBAR), or mechanical resonances.

Tunable Microelectromechanical Filters that Exploit Parametric Resonance

Background: This paper describes an analytical study of a bandpass filter that is based on the dynamic response of electrostatically-driven MEMS oscillators. Method of Approach: Unlike most mechanical and electrical filters that rely on direct linear resonance for filtering, the MEM filter presented in this work employs parametric resonance. Results: While the use of parametric resonance improves some filtering characteristics, the introduction of parametric instabilities into the system does present some complications with regard to filtering. Conclusions: The aforementioned complications can be largely overcome by implementing a pair of MEM oscillators with tuning schemes and some processing logic to produce a highly effective bandpass filter. [DOI: 10.1115/1.2013301]
3 Employing Parametric Resonance for Filtering

As noted in the introduction, one of the distinct advantages of the MEM filters presented in this work is that they exhibit nearly ideal stopband rejection and an extremely sharp response rolloff. The basis for this is that the proposed filters are composed of parametrically excited oscillators, rather than a chain of directly excited oscillators. The parametric excitation is generated by applying the fluctuating voltage signal across sets of noninterdigitated comb drives, as shown in Figs. 2(a) and 2(b), which produce a time dependent electrostatic stiffness through the resulting fringing electric fields [14]. As a result of this excitation, the oscillators remain quiescent until the frequency reaches the parametric resonance instability zone, whereupon they exhibit a rapid increase in response amplitude [9]. The resonant response amplitude is limited by nonlinearity, in contrast with the usual linear resonance, wherein damping limits the amplitude. While the presence of this phenomenon has some desirable features from a filtering point of view, the introduction of parametric instability into the system does present some problems, due to the intrinsic nature of the resulting resonant response. As described in more detail in the following sections, these difficulties include:

- The bandwidth and center frequency of the passband, that is, the center and width of the unstable frequency zone, depend on the amplitude of the excitation (input) signal.
- Nontrivial responses can exist outside of the passband, due to hysteresis.
- There is a nonlinear relationship between the system’s input and output.
- Higher order resonances can occur.

Fortunately, most of these deficiencies can be overcome by exploiting the tunable nature of MEMS, in conjunction with some logic implementation, as presented below.

4 Dynamics of a Parametrically Excited MEMS Oscillator

To provide a basic understanding of the proposed bandpass filters, their benefits, and the difficulties that must be overcome, consider the response of a single degree of freedom MEM oscillator, such as the one shown in Fig. 2(a) and Fig. 3. This device, similar to those considered in [14,15] and [4], consists of a shuttle mass, namely the oscillator’s backbone “B,” anchored to ground by four folded beam springs “S,” and excited by an oscillating voltage applied to sets of noninterdigitated comb drives “N.” The equation of motion for this oscillator can be expressed as [16]:

\[ m \ddot{x} + c \dot{x} + F_r(x) + F_{es}(x,t) = 0, \]  

where \( m \) represents the mass of the shuttle, \( c \) the damping coefficient (derived experimentally through logarithmic decrement methods), and \( F_r(x) \) the elastic restoring force from the springs, which is accurately modeled by a cubic function of displacement:

\[ F_r(x) = k_1x + k_3x^3, \]  

which is, in general, mechanically hardening (\( k_3 > 0 \)). The electrostatic driving and restoring forces, \( F_{es}(x,t) \), are produced by two independent noninterdigitated comb drives, as shown in Fig. 2(b). (The motivation for using two sets of comb drives will be
Table 1 Nondimensional parameter definitions

<table>
<thead>
<tr>
<th>Definition</th>
<th>Nondimensional parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \dot{y} = d(\tau) )</td>
<td>Scaled Time Derivative</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td></td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>Scaled Damping Ratio</td>
</tr>
<tr>
<td>( \epsilon V_0 = r_3 V_0^2 + r_4 V_0^4 )</td>
<td>Linear Electrostatic Stiffness Coefficient</td>
</tr>
<tr>
<td>( \epsilon A_x = r_3 A_x^2 )</td>
<td>Linear Electrostatic Excitation Amplitude</td>
</tr>
<tr>
<td>( \Omega = \omega / \omega_0 )</td>
<td>Nondimensional Excitation Frequency</td>
</tr>
<tr>
<td>( \lambda = k x_0^2 / k_1 )</td>
<td>Nonlinear Mechanical Stiffness Coefficient</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Nonlinear Electrostatic Stiffness Coefficient</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Nonlinear Electrostatic Excitation Amplitude</td>
</tr>
</tbody>
</table>

Note that this equation is not the typical nonlinear forced Mathieu equation, due to the presence of the parametric excitation acting on the cubic term, which has a significant effect on the oscillator’s response, as discussed in Sec. 6. The linear stability of the system is unaffected by these nonlinear effects, however, and the averaged equations (derived in Sec. 6) allow one to analytically approximate the instability zone for the trivial response. This is shown in Figs. 4(a) and 4(b) in terms of the \( V_A - \Omega \) parameter space, that is, the physical excitation amplitude (the fluctuating voltage amplitude) versus the normalized excitation frequency.
Note that the accuracy of the analytical predictions has been verified by results from simulations of the full equation of motion, from which the stability boundary is determined by examining the behavior (growth or decay) of small perturbations from the trivial response. Some important features of this stability diagram are considered next.

Due to the nature of the filtering approach described in this work, the instability zones depicted in Fig. 4 are of particular interest. First, note that the boundaries of these zones are curved, which is due to the fact that the excitation amplitude is proportional to the square of the ac voltage amplitude. A number of other observations are now described, which follow from well-known results for systems exhibiting parametric resonance [17]. Inside the “wedge of instability” the trivial response is unstable, which results in a nonzero response that is dictated by the nonlinearities present in the system. Outside of the boundary the trivial response is stable. However, this does not ensure a zero response outside the wedge, since the system can have multiple possible stable steady states, and nontrivial stable responses can occur outside the instability zone. This leads to potential hysteresis in the response, which is highly undesirable in a filter. Another characteristic of this instability is that the base of the instability wedge originates at the nondimensional frequency of 2, and therefore filtering takes place at twice the natural frequency of the oscillator, that is, twice that of filters that utilize direct excitation. Also, the height of this base depends in a very sensitive manner on the system damping, which arises primarily from aerodynamic and structural dissipation effects in the oscillator. This implies that there is a critical ac voltage input required for the oscillator to work properly. This, however, is of minimal concern since the oscillators typically act in an environment with extremely low damping (near vacuum for testing) where quality (Q) factors can range into the thousands. Also, the input (excitation) signal can be amplified, if needed, to attain the critical excitation amplitude. Finally, the amplitude-dependent nature of the stability boundary should be noted. A direct result of this is that the system’s bandwidth and center frequency depend on the excitation amplitude, some of the primary drawbacks mentioned above. These undesirable response features are overcome by the linear and nonlinear tuning schemes described in the next two sections, combined with the logic implementation presented in Sec. 7.

### 5 Manipulation of the Amplitude Dependence of the Stability Boundary

Figure 4 shows that the oscillator’s activation frequency (where the trivial solution undergoes a stability change) is dependent on \( V_A \), the amplitude of the ac voltage input. As such, when an un-tuned oscillator is employed as a filter, its bandwidth will be dependent on the excitation amplitude. However, it is possible to partially negate this effect through the implementation of a specific tuning scheme, namely, one in which the natural frequency of the oscillator, \( \omega_n \), is made to be dependent on \( V_A \) through a tuning of the linear electrostatic stiffness coefficient. This is accomplished by selecting the dc voltage in one set of combs to be dependent on \( V_A \), which is the amplitude of the ac voltage that acts on the other set of combs. A description of the tuning scheme and its attendant consequences are presented here. The theoretical predictions described here are based on the averaged equations, which are presented in the following section.

To begin, we note that the signal to be filtered is the ac signal, and that the dc signal has been introduced solely for this tuning task. The dc voltage is to be dictated by the amplitude of the ac signal, as follows. A designer-specified constant of proportionality \( \alpha \) is introduced that relates the amplitudes of the dc and ac voltage amplitudes according to

\[
V_0 = \alpha V_A.
\]

which results in the redefined parameters given in Table 2. Substituting these parameters into Eq. (9) results in a revised equation of motion, given by,

\[
z'' + z = -\epsilon [2\zeta z'' + \omega_n^2 (\rho + \cos(\Omega t)) + \zeta' (\chi + r_z + \lambda_3 \cos(\Omega t))],
\]

wherein a new tuning parameter, \( \rho \), is introduced that relates the linear electrostatic stiffness coefficient to the linear excitation amplitude, according to

\[
\rho = \frac{V_1}{V_A} = 1 + \frac{\rho_0 a^2}{r_{1A}}.
\]

The parameter \( \rho \) represents the net effect that the ac amplitude, expressed in terms of \( \lambda_3 \), has on the natural frequency of the oscillator. Specifically,

\[
\omega_n = \sqrt{1 + \epsilon \rho \lambda_1} = \sqrt{1 + \epsilon V_1}.
\]

The instability zone can thus be distorted by the variation of \( \rho \), since this produces a change in the linear natural frequency in a manner that is dependent on the input amplitude, \( V_A \), through \( \lambda_3 \). This results in rotation of the wedge of instability, away from the nominal case shown in Fig. 4(b), as shown in Figs. 5(a) and 5(b). In particular, by selecting \( \rho > 0 \) the wedge will rotate clockwise, and by selecting \( \rho < 0 \) the wedge will rotate counterclockwise. Note that one has the ability to set \( \rho \) by designing combs with the desired linear electrostatic characteristics and then selecting \( \alpha \) accordingly. Using perturbation calculations (outlined in Sec. 6), it can be shown that when \( r_{1A} > 0 \), by selecting \( \rho = 1/2 \) the left stability boundary of the wedge becomes essentially vertical, as shown in Fig. 5(a), and, similarly, by selecting \( \rho = -1/2 \) the right stability boundary becomes nearly vertical, as shown in Fig. 5(b) [4]. Higher order perturbation approximations can be used to improve this verticility, however, the resulting improvement in performance is minimal, so nominal values of \( \rho = \pm 1/2 \) are used in this work. Note that the opposite trends of those described here, in terms of the direction of the wedge rotations, occur when \( r_{1A} < 0 \).

The aforementioned verticality, achieved by selecting \( \rho = \pm 1/2 \), has the distinct advantage that it renders one of the oscillator’s activation frequencies to be amplitude independent and, as such, makes it act essentially like a high or low pass switch. In particular, for \( r_{1A} > 0 \), by selecting \( \rho = 1/2 \), the oscillator will act as a high pass switch, and by selecting \( \rho = -1/2 \) a low pass switch is achieved.

While this tuning provides a solution to one difficulty, others remain. We now turn to the possibility of nontrivial responses occurring outside of the instability zone, due to nonlinear hysteretic effects.

### 6 Nonlinear Tuning and Response Conditioning

As mentioned, one deficiency of the tuned oscillators presented above is that they have the propensity to feature nonzero response amplitudes outside of the wedge of instability, due to the presence of nonlinearities in the system. However, the flexibility of the comb drives in these MEM devices allows one to adjust the system nonlinearities through electrostatic forces. Specifically, as de-

### Table 2 Redefined nondimensional parameters [4]

<table>
<thead>
<tr>
<th>Definition</th>
<th>Nondimensional parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \epsilon V_A = \left( \frac{r_0 a^2 + n_i}{k_l} \right) V_A^2 )</td>
<td>Linear Electrostatic Stiffness Coefficient</td>
</tr>
<tr>
<td>( V_1 = \frac{x_0 a^2}{k_l} V_A^2 )</td>
<td>Nonlinear Electrostatic Stiffness Coefficient</td>
</tr>
</tbody>
</table>

\( V_0 = \alpha V_A \),

\( \rho = \frac{V_1}{V_A} = 1 + \frac{\rho_0 a^2}{r_{1A}} \),

\( \omega_n = \sqrt{1 + \epsilon \rho \lambda_1} = \sqrt{1 + \epsilon V_1} \),

426 / Vol. 127, OCTOBER 2005 Transactions of the ASME
scribed in [14] and [16], the cubic nonlinearity produced by
electrostatic effects in the comb drives can be tuned such that the
overall nonlinearity exhibits hardening, softening, or mixed
hardening/softening characteristics. This is verified by considering
the effective nonlinearity of the MEM oscillator, as determined
through a perturbation analysis, as follows.

Equation (11) is converted to the correct form for the applica-
tion of averaging by employing the standard transformation to
amplitude and phase coordinates, specifically,

\[
\tilde{z}(\tau) = \alpha(\tau) \cos \left( \frac{\Omega \tau}{2} + \phi(\tau) \right)
\]

and

\[
\tilde{z}'(\tau) = -\alpha(\tau) \frac{\Omega}{2} \sin \left( \frac{\Omega \tau}{2} + \phi(\tau) \right).
\]

In order to capture the response near the parametric resonance, an
excitation frequency detuning parameter \( \sigma \) is introduced, defined by,

\[\Omega = 2 + \varepsilon \sigma.\]

Equation (11) is transformed to the amplitude/phase coordinates,
\( a(\tau) \) and \( \phi(\tau) \), and the resulting equations governing the dynamics
of these variables are averaged over \( 4\pi/\Omega \) in the \( \tau \) domain [4].
This results in the averaged equations, which are given by,

\[a' = \frac{1}{8} \varepsilon e \left[ -8\zeta + (2\lambda_1 + a^2\lambda_3) \sin(2\phi) \right] + O(e^2)\]

Note that the presence of the nonlinear electrostatic parametric
excitation \( \lambda_3 \) leads to a complicating term in the averaged
equations (as compared to the usual nonlinear Mathieu equation). Con-
sequently, the nontrivial steady state solutions of the system take
on a quite complicated form, unless some approximations are
used.

Since the characteristic form of the nonlinearity (e.g., harden-
ing, softening, etc.) is the critical feature of the response for cur-
rent purposes, one can assume zero damping in order to simplify
the analysis. If one examines the steady state responses in this
case, it is found that the response has nontrivial branches with
amplitudes given by,

\[
\tilde{a}_1,2 = \frac{4\sigma - 2\lambda_1(2\rho - 1)}{3(\chi + \nu_3) - 2\lambda_3}, \quad \tilde{a}_3 = \frac{4\sigma - 2\lambda_1(2\rho + 1)}{3(\chi + \nu_3) + 2\lambda_3},
\]

and in certain cases additional nontrivial branches exist with am-
plitudes given by,

\[\tilde{a}_3 = \frac{-2\lambda_1}{\lambda_3},\]

the latter of which appear only in parameter ranges outside of
those considered here (a forthcoming paper describes these solu-
tions in detail [18]). The signs of the terms under the square root
sign determine the frequency ranges over which these branches
are real, and therefore physically meaningful. In fact, it is easily
seen that by selecting \( \rho = \pm 1/2 \) one can render the frequency (that
is, the \( \sigma \) values) where each branch appears to be independent of
the input amplitude \( \lambda_1 \). It is also interesting to note the role played
by the nonlinearities, which differ on the two branches. To de-
scribe these effects, effective nonlinear coefficients \( \gamma_1 \) and \( \gamma_2 \) are
introduced, which dictate the hardening or softening behavior of
the branches, as follows,

\[\gamma_1 = 3(\chi + \nu_3) - 2\lambda_3, \quad \gamma_2 = 3(\chi + \nu_3) + 2\lambda_3.
\]

Tuning the oscillator’s electrostatic coefficients such that both
\( \gamma_1 \) and \( \gamma_2 \) are less than zero results in an oscillator with the usual
softening characteristics. Likewise, tuning the coefficients such
that both \( \gamma_1 \) and \( \gamma_2 \) are greater than zero, results in an oscillator
with typical hardening characteristics. Clearly mixed softening/
hardening characteristics are possible as well, the details of which
are left for future investigations. Note, however, that since the
nonlinear coefficients are related to the input voltages (see Table 1
for \( \chi \) and \( \lambda_1 \), and Table 2 for \( \nu_3 \)), the nonlinear character of the
response may change as the input amplitude varies. This must be
accounted for in the designs, and may limit the range of allowable
ac input voltages for the resulting filter [18].

The flexibility in selecting the nature of the nonlinearity is very
useful for limiting the existence of nonzero solutions outside of,
or at least on one side of, the instability zone. In particular, by
specifying a hardening nonlinearity for a high pass switch \( \rho = 1/2 \), for \( r_{1A} > 0 \),
nontrivial responses below the activation fre-
quency can be avoided. Similarly, by specifying a softening non-
linearity for a low pass switch \( \rho = -1/2 \), for \( r_{1A} > 0 \), nontrivial
responses above the activation frequency can be avoided.

This nonlinear tuning can be achieved through careful design of
the comb drives. As alluded to earlier, precise values of the effec-
tive nonlinear coefficients are not required, since one simply
needs to maintain a particular sign of the effective nonlinearities,
so that hardening or softening persists, even in the face of the

that the switch is good at one frequency, but the other instability boundary is still amplitude-dependent, and the response will exhibit hysteresis. However, two such oscillators, tuned to act as amplitude independent switches at nearby frequency thresholds, have great potential for such use. In particular, it is possible to create a highly effective bandpass filter through the implementation scheme shown in Fig. 7.

The idea is to generate a bandpass filter which features a center passband frequency of $\Omega_0$ and a bandwidth of $\Delta \Omega_0$, where $\Delta$ is defined to be a small parameter which describes the bandwidth as a percentage of the center frequency (see Fig. 1). This parameter may also be used to define an “effective quality ($Q$) factor,” where $Q_{\text{eff}}$ is independent of system damping, as follows:

$$Q_{\text{eff}} = \frac{1}{\Delta}.$$  \hspace{1cm} (23)

To achieve this design, two oscillators are required. One is a high pass switch (with $\rho=1/2$ for $r_{A>0}$) that has been nonlinearly tuned such that it exhibits a hardening nonlinearity, which will henceforth be designated as “H.” The other is a low pass switch (with $\rho=-1/2$ for $r_{A>0}$) that has been nonlinearly tuned such that it exhibits a softening nonlinearity, which will henceforth be designated as “L.” In addition, both the L and H oscillators must have their linear mechanical frequencies tuned such that the base points of their respective wedges of instability are slightly shifted from $\Omega_0$, so that a passband is created. This can be achieved by designing the oscillators so that their zero-voltage (i.e., purely mechanical) linear instability threshold frequencies are as follows: for the L oscillator the threshold is selected to be $\Delta \Omega_0/2$ above $\Omega_0$, and for the H oscillator the threshold is selected to be $\Delta \Omega_0/2$ below $\Omega_0$. With these, the oscillators tuning is complete. A summary of the required tuning conditions for the two oscillators is given in Table 3.

Once the two oscillators have been tuned in accordance with the conditions set forth in Table 3, they are ready for implementation in the filter system presented in Fig. 7. This system is designed to function as follows. A harmonic input signal, $R$, of the form $R = V_A \cos(\omega t)$ is supplied to the system. This signal travels to a signal conditioner (SC) that produces an excitation signal appropriate for the oscillators’ comb drives, namely the square-rooted input described in Sec. 4. This signal is then used to drive both the H and L oscillators, and is also provided to the block designated $F_3$. This block represents an ac to dc converter that produces the amplitude of $R$, namely $V_A$ (which is monotonically related to $V_D$), or some proportion thereof. This dc signal is sent to the oscillators, where it is used to drive the two resonators, and to tune each oscillator through its linear tuning parameters, $\rho = \pm 1/2$ (recall that the $\rho$ tuning is set by $\alpha$, which sets $V_D$ in relation to $V_A$). Each oscillator, acting as described in previous sections, filters the provided input signal and acts as a switch, producing a zero (in practice, the noise floor) or oscillatory response, depending on the

7 Creating a Bandpass Filter

While the tuning schemes presented above do an excellent job of conditioning the response of an individual oscillator, they result in frequency-dependent switches, not a bandpass filter. Also, note that the instability wedges have been rotated as desired from the averaged equations for general sets of parameters in order to achieve overall softening over a reasonable range of frequencies.
frequency of the excitation signal. The respective signal from each oscillator is then sent to another block designated $F_2$ which converts the signal into a constant voltage, or to a digital signal. For example, the block may produce a 0 when the oscillator’s output is zero (noise floor) and 1 when the output is oscillating. The signal from each $F_2$ block then proceeds to an AND junction, which provides a nonzero signal to the enabling block $P$ only when the frequency of the input signal falls within the desired bandwidth. If the enabling device $P$ receives a nonzero signal, it allows the original filter input to pass unimpeded, otherwise, it blocks the signal. The result is a bandpass filter with ideal stopband rejection and optimal rolloff in its frequency response.

To verify the operation of the filtering scheme presented in Fig. 7, numerical simulations of the system were carried out using Simulink™. As Fig. 8 shows, the results for a bandpass filter designed with an effective quality factor of 500 are essentially as expected. The filter bandwidth is nearly amplitude independent (note the horizontal scale), and could be made even more so by improving the tuning parameter, $\rho$. In addition, the attenuation outside of the passband is absolute, which verifies that the filter’s stopband rejection is ideal.

8 Conclusions

Filtering based on parametric excitation has some very attractive features, as summarized in Fig. 8. For the implementation considered here, the only obvious drawback of note is that a damping-dependent critical ac excitation amplitude is required for operation (which can be addressed by restricting the input voltage to a specified range). Other potential concerns for implementation of such a filter include: the fact that the insertion loss of the physical system cannot yet be quantified, since this will depend on the hardware implemented for the filter; higher order resonances may appear in the system, which will lead to nontrivial responses well away from the passband; and design robustness issues may arise, for example, the required accuracies of the linear tuning strategy that rotates the wedge, temperature sensitivity, etc.. Also of interest are the transient behavior and the phase response of the parametrically-excited oscillators in question. In particular, the settling time required for an oscillator to reach steady state and the steady-state phase characteristics need further investigation. Preliminary simulation results indicate that the oscillators described here exhibit settling times comparable to those of linear oscillators with the same level of damping. However, due to the frequency dependence of the settling time in parametrically-excited systems, this result is valid only in a frequency domain near the passband center frequency, since the system’s time constant approaches infinity at the stability boundaries, where bifurcations occur. The phase behavior of the filter can be easily quantified from the averaged equations. These issues are currently being addressed in ongoing investigations, which include experimentation.

While this paper focused on bandpass filters, it is worth noting that an equally ideal band gap filter can be produced by simply changing block $P$ such that it enables with a zero amplitude signal instead of a nonzero amplitude signal. It is also expected that these parametric-based frequency switches can be used to develop high and low pass filters. The ultimate goal of this line of work is to achieve fully functional filters wherein the parametrically-excited MEM oscillators and the associated circuitry are integrated into a single chip.
APPENDIX A: Design Parameters

<table>
<thead>
<tr>
<th>( \rho = 1/2 ) Oscillator</th>
<th>( \rho = -1/2 ) Oscillator</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_1 = 10 \mu N/m ) (69 kgf/ft)</td>
<td>( k_1 = 10 \mu N/m ) (69 kgf/ft)</td>
</tr>
<tr>
<td>( k_2 = 0.05 \mu N/m^2 ) (3.2E8 lb/ft^2)</td>
<td>( k_2 = 0.05 \mu N/m^2 ) (3.2E8 lb/ft^2)</td>
</tr>
<tr>
<td>( x_0 = 1 \mu m ) (3.2E-6 ft)</td>
<td>( x_0 = 1 \mu m ) (3.2E-6 ft)</td>
</tr>
<tr>
<td>( m = 4.052E^{-10} ) kg (8.9349E-10 lbm)</td>
<td>( m = 4.052E^{-10} ) kg (8.9349E-10 lbm)</td>
</tr>
<tr>
<td>( \zeta = 0.01 )</td>
<td>( \zeta = 0.01 )</td>
</tr>
<tr>
<td>( r_{11} = 2.00E-3 \mu N/m V^2 ) (1.37E-4 lb/ft V^2)</td>
<td>( r_{11} = 2.00E-3 \mu N/m V^2 ) (1.37E-4 lb/ft V^2)</td>
</tr>
<tr>
<td>( r_{22} = -5.00E-4 \mu N/m V^2 ) (-3.43E-5 lb/ft V^2)</td>
<td>( r_{22} = -5.00E-4 \mu N/m V^2 ) (-3.43E-5 lb/ft V^2)</td>
</tr>
<tr>
<td>( r_{12} = 1.00E-3 \mu N/m V^2 ) (6.37E-6 lb/ft V^2)</td>
<td>( r_{12} = 1.00E-3 \mu N/m V^2 ) (6.37E-6 lb/ft V^2)</td>
</tr>
<tr>
<td>( r_{21} = -2.00E-4 \mu N/m V^2 ) (-1.59E-5 lb/ft V^2)</td>
<td>( r_{21} = -7.50E-4 \mu N/m V^2 ) (-1.59E-5 lb/ft V^2)</td>
</tr>
</tbody>
</table>

Acknowledgements

This work is supported by the AFOSR under Contract No. F49620-02-1-0069. S.W.S. would like to thank the Mechanical Engineering Department at the University of California, Santa Barbara (UCSB) for their hospitality during a sabbatical visit, during which the work for this paper was completed. In addition, the authors are grateful to Professor Jeff Moehlis and Barry DeMartini of UCSB for useful input on the analysis of the oscillator equation and the preliminary designs of the MEMS oscillators, respectively.

References