Abstract—Robust feedback control design of an ultra-sensitive, high-bandwidth, tunneling accelerometer is reported. The control design uses the $H_\infty$ loop-shaping technique to shape the loop to achieve high disturbance rejection, noise attenuation, and robustness to parameter variations. While the design was performed on a linearized plant, nonlinear simulations of the tunneling accelerometer device demonstrate that the accelerometer can accurately sense forces over its control bandwidth even when its parameters have been perturbed away from their nominal values that were used in the control design.

I. INTRODUCTION

High-bandwidth accelerometers are needed in many applications including seismic activity detection, inertial sensors (aerospace), and tracking devices. Micro-machined tunneling accelerometers are appealing for their high sensitivity [1], [3], [4], [5], however, noise [10] and reliability have been two issues hindering their long-term stability and use. Reliability is being addressed through advances in MEMS fabrication. By utilizing Single Crystal Silicon processes and careful packaging techniques reliability can be improved [2]. A key component of reliability is device robustness. Accelerometer parameters, particularly those associated with the tunneling component, are invariably uncertain and drift during the operation of the device. Designing for robustness is therefore essential for achieving both reliability and noise attenuation.

In this paper, we have designed a robust controller for a silicon micromachined tunneling accelerometer using $H_\infty$ loop-shaping methods [11], [12]. Previous control strategies proposed for microscale tunneling accelerometers [2], [3], [6] have either used PID control and did not incorporate uncertainty in the design process, or have used the $\mu$ synthesis resulting in controllers of very high order. We show that the robust $H_\infty$ loop-shaping offers an attractive alternative design approach for this application that offers relative simplicity, connection with classical control design methods, and relatively low order controllers. We demonstrate, in simulation, that the design is very effective over a large bandwidth while being robust to parameter variations. Experimental implementation of this controller onto a MEMS accelerometer is underway, and we hope to present experimental results in the final publication.

In accordance with the theory of quantum mechanics, electron tunneling can be experimentally observed when the gap between a pair of clean electrodes with a voltage bias $V_0$ is sufficiently small. The resulting tunneling current is extremely sensitive to the gap separation distance. Indeed the tunneling current varies exponentially according to its gap. This fact is exploited by affixing an electrode to a suspended proof-mass that can move towards a second fixed electrode (the tunneling tip). See Figure 1. As the distance between the two electrodes changes, so does the tunneling current. Thus, the tunneling current can be used as a very sensitive measure of the displacement of the proof mass, close to the tunneling distance, which for this application is typically of the order of a nanometer (10 Å). By applying a voltage controlled electrostatic force, the position of the proof mass may be actuated. Using the tunneling current as a feedback signal, the position of the proof mass may be controlled to remain at a fixed distance from the tunneling tip. This distance must be maintained through the feedback control system despite the presence of external forces that tend to move it away from the equilibrium position. If good external force rejection is achieved, the control signal itself may be used as an accurate measure of such forces. In the case of the tunneling accelerometer, this would be captured by the voltage feedback signal supplied to the electrostatic actuator.

II. ACCELEROMETER MODEL

In this section, we describe the mathematical model of the tunneling accelerometer. At rest, the proof mass is 1200 Nm away from the tunnel tip. We let $x$ be the displacement of the proof mass from the rest position. Newton’s law gives the motion of the proof mass:

$$m \frac{d^2 x}{dt^2} + c \frac{dx}{dt} + kx = F_e + F_d$$

where $m = 5.2 \times 10^{-9}$ is the mass, $c$ is the damping coefficient, and $k$ is the spring constant. The forcing terms consist of the electrostatic force $F_e$, and the external disturbance.
force $F_d$, whose measurement is the desired output of the device.

The electrostatic force exerted by the comb drive is given by

$$F_e = N \frac{\varepsilon_0 h}{g} V^2,$$

where $N$ is the number of fingers, $\varepsilon_0$ is the dielectric constant of air, $h$ is the depth of each plate, $g$ is the gap between the parallel electric plates, and $V$ is the applied bias across the plates of the comb drive.

Letting $\omega_0 = \sqrt{\frac{k_0}{m}}$ and $Q = m \frac{\omega_0}{\varepsilon_0}$, we have

$$\frac{d^2 x}{dt^2} + \frac{\omega_0}{Q} \frac{dx}{dt} + \omega_0^2 x = \frac{1}{m} [F_e + F_d].$$

The particular design considered has a proof mass of 5.2 micrometers with a spring constant $k = 6.2$ N/m. The measured value for the natural frequency $\omega_0$ is 5.5 kHz, and $Q = 20$. The number of fingers $N = 500$, the dielectric constant of air is $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m, and the depth of each plate $h = 1.3 \times 10^{-5}$ m. The value of the gap between the parallel plates $g$ has been designed so that when the tip-proof mass rest gap is $d = 1.2 \mu m$ the actuator can close this gap with a bias voltage of 15V. Using a nominal stiffness coefficient of $k_0 = 6$ N/m, and equating $k_0 d$ to $F_e$, we get that the designed value of $g = 1.8 \mu m$.

### A. Scaling

With the above design parameters, the problem is in need of scaling. We begin by scaling time $t \to 10^{-3} t$ so that the new time variable has units of milliseconds (the frequency variable will also be krad/s). Furthermore, we scale the displacement variable $x \to 10^{-9} x$ and the new $x$ variable has units of nanometers. Combining the two scalings together we have

$$\frac{d^2 x}{dt^2} + 10^{-3} \omega_0 \frac{dx}{dt} + 10^{-6} \omega_0^2 x = \frac{10^3}{m} (F_e + F_d).$$

As stated here, the forces $F_e$ and $F_d$ are in Newtons. It is convenient to use units of nano-Newton to measure these forces, which we do. With these units we have

$$\frac{d^2 x}{dt^2} + 10^{-3} \omega_0 \frac{dx}{dt} + 10^{-6} \omega_0^2 x = \frac{10^{-6}}{m} [F_e + F_d],$$

where $F_e = 10^9 N \frac{e_{air} V^2}{g}$ (nano-Newton). In the frequency domain we have: $X(s) = G(s)(F_e(s) + F_d(s))$, where

$$G(s) = \frac{10^{-6}}{s^2 + 10^{-3} \frac{\omega_0}{\varepsilon_0} s + 10^{-6} \omega_0^2}.$$

This is expressed in block diagram form in Figure 2, where $E = 10^9 N \frac{e_{air} V^2}{g}$.

### B. Tunneling Current Measurement and Feedback

In the setup of the accelerometer device, the value of $x$ is measured only indirectly through measuring the tunneling current $i$. This current is proportional to the tunneling bias across the tunneling electrode gap and depends exponentially on the tunneling gap distance $x_g$:

$$i \propto V_0 e^{-\alpha \sqrt{x_g}},$$

where $\alpha$ is the tunneling constant, $h$ is the effective height of the tunneling barrier. For the present design, we have

$$i = 20 \frac{e^{-23.719(d-x)}}{x}$$

where $d = 1200$ nm is the distance from the proof-mass and the tunneling tip when the system is at rest. The desired setpoint distance between the proof-mass and the tunneling tip is set to be 1 nm. The proof-mass displacement to be tracked is therefore 1199 nm. At this distance the bias voltage is selected so that the tunneling current is 1 nA. The control objective is now to track a 1 nA tunnel current. The closed-loop setup of this problem is shown in figure 2. An integrator is used so that the current setpoint is tracked robustly. It should be pointed out here that due to the possibility of significant drift in the tunneling parameters the measured tunneling current may not be used to solve back for the displacement. However, tracking a specific distance is not necessary. Instead what is important is that a constant distance be maintained, even if that distance cannot be determined exactly. In essence, the tunneling current will be regulated because it can be measured, but the corresponding displacement, while regulated, has an uncertain value.

### III. CONTROL DESIGN

The objective of the control is to stabilize the system about the tracked equilibrium and to reject the external disturbance $F_d$ despite uncertainty in the tunneling current/displacement relation. The system is linearized about the tracked equilibrium Figure 3. The linearized tunneling current is $K_{tunnel} \delta x$ where $\delta x$ is the deviation from the
operating distance corresponding to 1 nA of tunneling current. This distance is equal to 1 nm if the tunneling current is indeed given by $i = 20e^{-23.719(d-x)}$. The linearized gain at that operating point is $K_{\text{tunnel}} = 10^9 \times 20 \times 23.719 \times e^{-23.719} = 23.719$ nA/nm. While this value serves as a nominal value, $K_{\text{tunnel}}$ there is considerable in its value.

As pointed out earlier, the tunneling current parameters are typically unknown exactly and their values will gradually drift. For this reason, and because different devices will have different nominal values for all their parameters (not just tunneling parameters), a central objective of the design is to achieve stability and disturbance rejection that is robust to variations in $K_{\text{tunnel}}$ away from the nominal value and to variations in the model parameters for $G(s)$.

We will use $H_\infty$ loop-shaping design [11], [12] to meet the design objectives. By rejecting the effect of the disturbance $F_d$ at the input of $G(s)$ robustly, we ensure that $\delta F_e$ follows $F_d$ over the frequencies where this rejection is achieved. In this case, the control signal $V^2$ necessary to achieve the rejection gives an accurate measure of the disturbance itself. This is the main principle of measurement of $F_d$. Clearly, the larger the frequency of the rejected disturbances, the larger the bandwidth of operation of the device. The limitation to the bandwidth ultimately comes from the measurement noise.

To meet the disturbance rejection requirements we shape the loop transfer function to be large over lower frequencies up to the control bandwidth. The plant is $P(s) = K_{\text{tunnel}}G(s)$. Figure 4 shows the plant frequency response and the desired loop shape $W(s)P(s)$ achieved by designing a pre-filter $W(s)$. The shaping filter is given by:

$$W(s) = \frac{(s+0.005)}{s(s+0.1)}.$$

Note that the weighting function includes an integrator. Since any stabilizing controller of $P(s)W(s)$ cannot cancel the pole at the origin, the final controller we get (which has $W(s)$ in its dynamics) will also include an integrator.

![Fig. 2. Tunneling Accelerometer Feedback Loop](image)

![Fig. 3. Tunnelling Accelerometer Linearized Feedback Loop](image)

![Fig. 4. Bode plot of the plant $P(s)$, the shaped plant $P(s)W(s)$, and $L(s) = P(s)W(s)K_s(s)$](image)

$$K(s) = 43.2096\frac{(s+0.005016)(s^2+5.059s+7529)}{s(s+0.1)(s^2+235.5s+35260)}$$

and the controller $K'(s) = \frac{1}{s}K(s)$. The designed loop gain $L(s) = G(s)W(s)K_s(s)$ is shown in Figure 4. For this design the gain margin is 12.322 dB and the phase margin is 71.3°. The bode plot of the plant $P(s)$ and $K(s)$ are shown in figure 5. The sensitivity function $S(s) = \frac{I-P(s)K(s)}{I}$ and the complementary sensitivity function $1-S(s)$ are plotted in figure 6. From the figure it is clear that disturbances between 0 and 10 Krad/s are well rejected.

IV. NONLINEAR SIMULATIONS

Using the designed robust controller, simulations of the nonlinear system were performed. An external force signal $F_d$ was generated and used to test the accelerometer. The external force and the response of the accelerometer variables to this force is shown in Figure 7.

It can be seen that the electrostatic force is the sum of two signals: a constant signal which is equal to the equilibrium value corresponding to a tunneling current reference step.
with no external forces applied, and a second signal with is the response solely to the external force $F_d$. Since the purpose of this force is to reject the $F_d$, its value follows $-F_d$. This force is related to the comb drive voltage-squared through multiplication by $E$ and therefore its value can easily be measured. As may be seen from the second plot, the nonconstant component of the electrostatic force gives a fairly accurate measure of the external force $F_d$. The sum of $F_d$ and $E_i$ is shown in Fig.7 (3rd plot). In the same figure, the fourth plot shows that the tunneling current is regulated to 1 nA. The fluctuations around this setpoint are virtually unaffected by the perturbations that were imposed.

The distance is effectively maintained around 1 nm with very little deviations.

To test the robustness of the system we first perturbed the bias $V_b$. The perturbed tunneling current relation becomes

$$i_p = 40e^{-23.719(d-x)}$$

which is 100% larger than the nominal value used in the design and first simulation. Next, the nominal proof-mass dynamics

$$G(s) = \frac{10^{-6}}{s^2 + 10^{-3} \frac{m}{Q_s}s + 10^{-6}\omega_0^2}.$$}

were perturbed to $G_p(s)$ by decreasing $\frac{m}{Q_s}$ by 50% and increasing $\omega_0^2$ by 50%. Figure 8 shows the bode plot of $G(s)$ and $G_p(s)$. With both of these changes in place, and using the same robust controller as before, we repeated the nonlinear simulations. The results are shown in Figure 9.

This figure shows that the $F_e$ signal continues to capture the $F_p$ force very accurately despite the large change its equilibrium value. Thus the performance of the device is virtually unaffected by the perturbations that were imposed. The tunneling current continues to be robustly regulated around its setpoint, while the distance of the proof-mass to the tunneling tip has slightly increased.
A robust controller has been designed for use in a tunneling accelerometer device. The design, which was achieved using $H_{\infty}$ loop-shaping approach, resulted in a system that achieves high performance over its bandwidth and is robust to expected parameter variations. Experimental implementation of this controller onto a MEMS accelerometer is currently underway. Experimental results will be reported when they become available.

REFERENCES


V. CONCLUSION

A robust controller has been designed for use in a tunneling accelerometer device. The design, which was achieved using $H_{\infty}$ loop-shaping approach, resulted in a...