

FIG. 1.

Fractal lifetimes in the transition to turbulence

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Several parallel shear flows, including plane Couette flow between flat plates, and Hagen–Poiseuille flow down a circular pipe, do not show a linear instability in the flow regime where the transition to turbulence is observed. Experiments¹ and numerical simulations^{2,3} indicate that the decay of a perturbation or its growth towards a turbulent state depends fairly sensitively on the initial conditions. And even if the flow swings up to a turbulent dynamics, it may not remain turbulent forever. It therefore becomes meaningful to study the time a trajectory remains turbulent as a function of Reynolds number and type and amplitude of initial perturbation. Analyzing in detail the dynamics for slightly different initial conditions is a challenge for full numerical simulations because of enormous computing times required. However, within a low-dimensional model designed to capture the dominant features of turbulent shear flows⁴ it becomes possible to study the intricate structures of the lifetime landscape.

The figures show the color-coded lifetimes as a function of initial amplitude and Reynolds number for the nine-mode model derived in Ref. 4. The blue colors reflect short lifetimes and can be found for low Reynolds numbers and small amplitudes. The dark red colors indicate lifetimes in excess of the maximal integration time of 1500. Figure 1 shows the

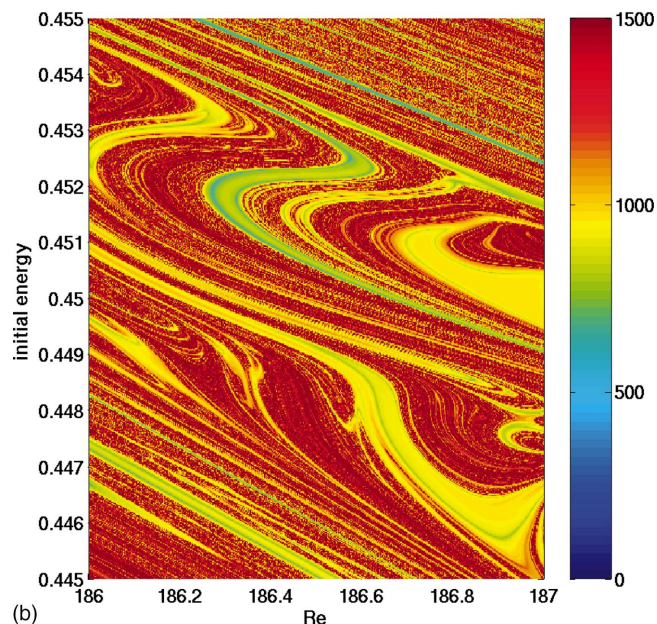
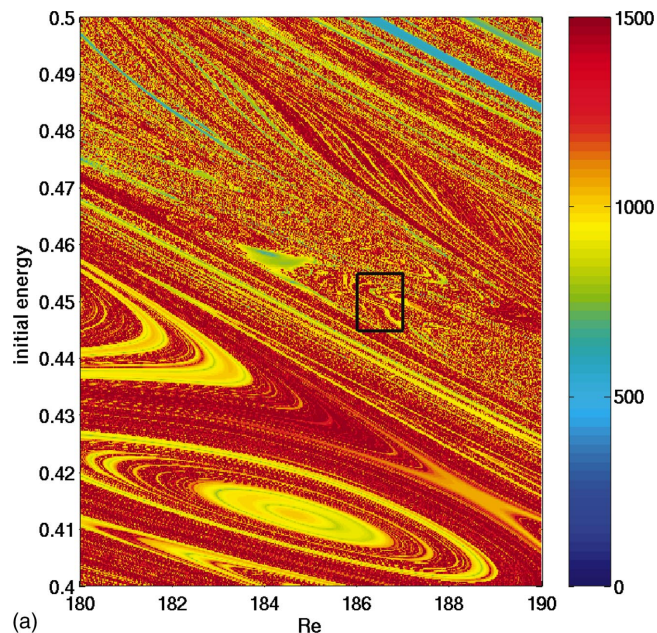


FIG. 2.

transition region and Figs. 2(a) and 2(b) show successive magnifications. Since the long lifetimes arise for initial conditions close to stable manifolds of the turbulent state, the folds in the lifetimes reflect folds in the manifolds. A more complete analysis of the model and its dynamical system features is given in Refs. 4 and 5.

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