1 Introduction

Gravity currents represent a ubiquitous phenomenon in nature and technology. They constitute primarily horizontal flows that are driven by hydrostatic pressure gradients due to variations in temperature or chemical composition. Examples of atmospheric gravity currents include sea breezes and thunderstorm outflows, while the Mediterranean and Red Sea outflows represent important oceanic gravity currents. Within the realm of technical applications, gravity currents may be encountered under a large variety of circumstances, including the heating and cooling of buildings, during tunnel fires, within water treatment facilities, as oil slicks on the ocean’s surface, or during CO2 sequestration in depleted oil reservoirs. A highly accessible introduction to the beautiful field of gravity current research can be found in the book by Simpson [1], while a more mathematically rigorous treatment of the topic is provided by Ungarish [2]. For readers interested specifically in environmental gravity-driven flows, the individual chapters in Ref. [3] summarize the state of the art and are well suited as entry points for carrying out research in this field.

Frequent gravity currents are driven by pressure gradients resulting from spatial variations in particle loading, such as in desert sand storms (“haboobs”) and powder snow avalanches [4]. Turbidity currents—derived from “turbid,” meaning “muddy”—represent an important class of such particle-driven flows [5]. They can be encountered in lakes as well as the ocean, where they are driven by the density difference between turbid water containing sand and/or clay, and clear ambient water. In freshwater reservoirs, they contribute to the loss of storage capacity over time, and in the world’s oceans they represent a key mechanism for transporting sediment from the continental shelves to the deep sea. Large turbidity currents can last for hours or even days, and they can propagate over vast distances in excess of \( O (1000 \text{ km}) \), such as along the North Atlantic Mid-Ocean Channel [6]. Their interaction with the seafloor via erosion and deposition is responsible for the formation of large-scale features such as submarine sediment waves, dunes, and canyons. Over geological time scales, the deposits from turbidity currents (turbidites) can reach enormous scales of up to \( O (10^6 \text{ km}^3) \), such as the Bengal Fan [7]. Under certain ambient conditions, the organic matter contained in the sediment may form hydrocarbons, so that sedimentary rock from turbidity current deposits plays an important role in oil and gas exploration [8]. From an engineering point of view, turbidity currents pose a significant hazard to submarine oil pipelines, wellheads, and telecommunication cables.

The gravity currents can form under such a wide variety of conditions and render them a particularly fascinating research topic. They can be associated with opposite ends of the Reynolds number spectrum (magma flows versus atmospheric currents), so that they are governed by different balances between inertial, viscous, and gravitational forces. They can be nonconservative in that their excess density varies with time (eroding or depositing turbidity currents), they can be Boussinesq or non-Boussinesq in nature (sea breezes versus powder snow avalanches), they can give rise to non-Newtonian dynamics (debris flows), and they can be linked to chemical reactions or to the preferential rejection of salt during the freezing of water. Especially, turbidity currents are multiscale in nature, as their large-scale evolution is closely tied to the microscale mechanisms of erosion and resuspension. Gravity currents can exist in ambient environments that exhibit velocity shear, such as in thunderstorm outflows [9], they can interact with a background density stratification, thereby triggering the formation of internal gravity waves [10], and their dynamics can be affected by complex topography [11]. Frequently, several of the above effects conspire to render the flow particularly complex, such as in certain types of snow avalanches, which may be non-Boussinesq, non-Newtonian, nonconservative, multiscale, and highly turbulent in nature. As a result, it is usually difficult to classify naturally occurring gravity-driven flows into neatly separated categories according to which of the many potentially relevant physical mechanisms may or may not be influential under a particular set of circumstances.

Various aspects of gravity current research have been reviewed in earlier articles. Hopfinger [4] provides an overview of powder snow avalanche research, while Rottman and Linden [12] summarize the basic scaling laws and force balances for idealized compositional gravity currents. Huppert [13] surveys a wide range of topics related to gravity-driven geophysical flows, while in Ref. [14] the same author focuses more exclusively on box models and shallow water equations for turbidity currents. In Ref. [15], he discusses aspects of dilute as well as concentrated particle-laden currents, along with dense granular flows. Kneller and Buckee [16] review theoretical approaches and experimental data for turbidity currents from a geological perspective. Parsons et al. [17] provide...
an overview over sediment gravity flows in the ocean, while the recent article by Meiburg and Kneller [5] reviews specifically turbidity currents and their deposits.

Partly as a consequence of this multitude of relevant flow regimes, a wide variety of modeling approaches for gravity and turbidity currents have evolved, spanning the entire range from dimensional analysis to high-resolution DNS. In the following, we will review these modeling approaches, with an emphasis on depth-resolving Navier–Stokes simulations.

Section 2 highlights several conceptual models of high Reynolds number gravity currents. While these models are frequently based on such simplifying assumptions as steady-state and inviscid flow, they nevertheless provide fundamental insight into the basic mechanisms driving such currents, and specifically into the scaling laws governing their front velocity. Section 3 briefly discusses the fundamental concepts behind depth-averaged modeling, as exemplified by box models and shallow water approaches. Section 4 reviews depth-resolving modeling approaches based on the full Navier–Stokes equations, including LES and RANS simulations. It furthermore highlights some areas of research where such simulations have contributed to our physical understanding in recent years, such as interactions between gravity currents and engineering installations or seafloor topography, non-Boussinesq gravity currents, and gravity currents propagating in stratified ambient. Section 5 identifies some of the current research challenges in the field and provides a brief outlook.

2 Conceptual Models: The Froude Number as a Function of the Current Height

Attempts to determine the front velocity of a gravity current as a function of its height and its excess density date back at least three quarters of a century, when von Kármán [18] introduced the idealized gravity current model shown in Fig. 1(a). He considered the flow in the reference frame moving with the current front, and invoked three main simplifying assumptions: (i) the flow is steady in this reference frame; (ii) the flow is inviscid; and (iii) the fluid inside the current is at rest. By applying Bernoulli’s law along the streamlines C-O and O-A, i.e., by assuming that the mechanical energy is conserved along these streamlines, he obtained for the Froude number

\[ F_h = \frac{U}{\sqrt{g' h}} = \sqrt{\frac{\gamma - 1}{\sigma}} \tag{1} \]

where \( U \) denotes the front velocity of the gravity current, \( h \) represents its height, \( g' = g(\rho_1 - \rho_2)/\rho_1 \) indicates the reduced gravity, and \( \sigma = \rho_2/\rho_1 \) refers to the density ratio.

Benjamin [19] objected to von Kármán’s analysis on the grounds that Bernoulli’s equation should not be assumed to hold along streamline O-A, due to the dissipation that occurs in this interfacial region as a result of the velocity shear between the current and the ambient. Benjamin instead considered a corresponding gravity current in a channel of finite depth \( H \), as shown in Fig. 1(b). By applying the same three simplifying assumptions as von Kármán, and also considering the pressure distributions far up- and downstream of the current front to be hydrostatic, Benjamin was able to write the conservation laws for mass and horizontal momentum flux as

\[ UH = U_2(H - h) \tag{2} \]

\[ p_C H + \rho_2 U^2 H = p_B H + \frac{1}{2} \sigma(\rho_1 - \rho_2) h^2 \tag{3} \]

\[ - g(\rho_1 - \rho_2) H h + \rho_2 U^2 (H - h) \tag{4} \]

For a given set of values for current thickness, channel height, and density ratio, the above relationships represent two equations for the three unknowns, \( U, U_2 \), and \( p_B - p_C \), so that one additional equation is required. To close the problem, Benjamin followed von Kármán’s approach and applied Bernoulli’s law; however, he effectively did so along the bottom wall C–B of the channel, rather than along the interface as von Kármán had done. For a current of fractional height \( z = h/H \), Benjamin thus obtained the Froude number

\[ F_{H,h} = \frac{U}{\sqrt{g' H}} = \sqrt{\frac{z(1 - z)(2 - z)}{\sigma(1 + z)}} \tag{5} \]

Note that the Froude number \( F_h \) based on the current height is related to the Froude number \( F_{H,h} \) based on the channel height by \( F_h = F_{H,h} \sqrt{\frac{z}{1 - z}} \).

By then applying Bernoulli’s equation along the top of the channel (D–E), Benjamin showed that an energy-conserving current is possible only for \( z = 1/2 \); currents with \( z > 1/2 \) require an input of energy to be realized. Several studies attempted to derive the Froude number by combining global energy considerations with the solution of a lock-release current, as reported and discussed by Yih [20], Shin et al. [21], and Ungarish [22, 23]. Such flows are not of the “steady-state” type considered here, and the details are beyond the scope of this brief outline.

In the context of Boussinesq gravity currents, Borden and Meiburg [24] show that invoking an energy closure assumption à la von Kármán and Benjamin becomes unnecessary if the conservation of vertical momentum is enforced, along with the conservation of mass and horizontal momentum. This approach bypasses the search for the “correct: energy closure entirely, as the conservation of energy or head loss arguments never enter the picture when calculating the front velocity. While there is no flow of vertical momentum into or out of the control volume \( BCD'E \) in Fig. 1(b), the importance of vertical momentum conservation nevertheless is immediately obvious: inside the control volume, the ambient fluid is first accelerated and then decelerated in the vertical direction, which affects the pressure profiles along the top and bottom walls. In turn, these profiles determine the pressure jump \( p_B - p_C \) across the current front, for which the need of an additional equation originally arose. Borden and Meiburg [24] show that the conservation of vertical momentum can easily be
accounted for by considering the linear combination of the differential versions of the steady-state, inviscid, horizontal, and vertical momentum equations, in the form of the dimensional Boussinesq vorticity equation

$$\mathbf{u} \cdot \nabla \omega = -g \frac{\partial \rho}{\partial x}$$  \hspace{1cm} (6)

where $\omega = \partial v/\partial x - \partial u/\partial y$ denotes vorticity, $\rho$ indicates the normalized density, and $x$, $y$, $u$, and $v$ represent the horizontal and vertical directions and velocity components, respectively. By integrating Eq. (6) over the control volume, we obtain a relation governing the total circulation around the control volume

$$\oint \mathbf{u} \cdot n \, ds = -g \frac{\partial \rho}{\partial x} \, dA$$  \hspace{1cm} (7)

Equation (7) states that the flow of vorticity into and out of the control volume is balanced by the baroclinic generation of vorticity inside the control volume. For a sharp interface, the area integral over the control volume is balanced by the baroclinic generation of vorticity alone. Up to this point, we have used the conservation of vorticity equation. Combining the vorticity conservation relationship Eq. (8) with the continuity Eq. (2) immediately produces

$$\begin{align*}
\frac{1}{2} U_2^2 &= g'h \\
\text{Combining the vorticity conservation relationship Eq. (8) with the continuity Eq. (2) immediately produces}
\end{align*}$$  \hspace{1cm} (8)

Borden and Meiburg [24] show that for Boussinesq currents this relationship between the Froude number and the current height results in better agreement with DNS simulation results regarding the vorticity flux than Benjamin’s relationship (5). We note that in the above analysis, the pressure jump $p_b - p_c$ across the current front has been decoupled from the problem of determining $U_1$ and $U_2$, which were evaluated from the conservation of mass and vorticity alone. Up to this point, we have used the conservation of horizontal momentum only in linear combination with the conservation of vertical momentum, i.e., as the vorticity equation. Consequently, if desired, the pressure jump $p_b - p_c$ across the current front can now be determined from the horizontal momentum equation, as shown in Ref. [24]. The decoupling of the pressure in the above analysis very much corresponds to employing the streamfunction-vorticity formulation of the Navier–Stokes equations, which allows for the numerical simulation of incompressible flow fields without having to explicitly calculate the pressure. We note that, by accounting for the conservation of mass, horizontal, and vertical momentum, the above analysis does not have to invoke any assumptions about energy conservation. Rather, individual terms in the energy equation can now be evaluated, so that the overall loss of energy can be calculated a posteriori, rather than assumed a priori. This modeling concept employing the conservation of vorticity holds considerable promise for gravity currents propagating under more complex conditions, such as into sheared ambients [27].

Most recently, the vorticity-based approach has been extended to the non-Boussinesq regime by Konopliv et al. [28], who show that for a density ratio $\sigma = \rho_2/\rho_1$ one obtains a Froude number of

$$F_{h,\sigma} = \sqrt{\frac{2a}{\sigma}} (1 - a)$$  \hspace{1cm} (10)

In the limit of small density contrasts $\sigma \approx 1$, so that the Boussinesq result is recovered. We note, however, that in the non-Boussinesq case the pressure no longer fully decouples from the equations for the velocity field. Other recent extensions of conceptual gravity current models address such issues as two-layer and linearly stratified ambients, nonrectangular cross sections, in- and outflow, as well as fluctuations about the average [29–33].

For both Boussinesq and non-Boussinesq currents, Fig. 2 compares the different model predictions for the Froude number as a function of the fractional current height $a$. Note that Froude number in the figure is defined in terms of the current height and it includes the density ratio factor, i.e., $F_{h,\sigma} = F_{h,\sigma} \sqrt{\sigma}$ and $F_{h,\sigma} = F_{h,\sigma} \sqrt{1/2 \sigma}$. For currents occupying half the channel height, both Benjamin and circulation models predict a Froude number of $1/\sqrt{2}$. The predictions by Benjamin’s model and the circulation model show small differences for $0 < a < 0.5$.

3 Depth-Averaged Models

It is useful to distinguish between continuous inflow gravity currents and those that originate from the release of a finite volume of dense fluid, such as lock–exchange flows. Under certain conditions, much insight into both classes of flows can be gained from simplified modeling approaches in conjunction with dimensional scaling arguments. Especially useful in this regard have been so-called box models for finite volume currents, and shallow water analyses for currents whose length greatly exceeds their depth. In the following, we will briefly discuss the basic concepts underlying these modeling approaches.

Under many conditions, gravity currents form due to the release of a finite volume of dense fluid, a process that can be modeled by the prototypical case of lock-release currents, cf. Fig. 3. This configuration has served as the basis of numerous experimental, theoretical, and computational investigations into the physical mechanisms governing gravity currents. Consider a rectangular channel of height $H$ and length $L$. The channel is filled with two fluids of different densities that are initially separated by a barrier. While the “lock” of height $d$ holds a fluid of density $\rho_1$, the ambient fluid has lower density $\rho_2$. This initial configuration causes a discontinuity of the hydrostatic pressure across the barrier, which sets up a predominantly horizontal flow once the barrier is removed. The case of $d = H$ is referred to as a “full depth current.” Here the denser fluid forms a negatively buoyant gravity current propagating rightward along the bottom of the channel, while the lighter fluid moves leftward along the top as a positively buoyant

![Fig. 2. Model predictions for the Froude number as a function of the fractional current height $a$ for gravity currents: Benjamin’s model (solid line), circulation model (dashed-dotted line), and von Kármán model (solid circle).](image-url)
current. When \(d < H\), we obtain a “partial depth current,” for which the left-moving buoyant current takes the form of a rarefaction wave or bore propagating along the horizontal interface between the two fluids.

Huppert and Simpson [34] demonstrated that under high Reynolds number conditions full depth lock-release flows go through a well-defined sequence of stages. During the initial “slumping phase,” the current front travels for about \(O (5 \sim 10)\) lock lengths at constant velocity. This phase frequently comes to an end when the reflected bore generated at the left wall catches up with the front. The current subsequently enters a second stage governed by the balance of gravity and inertia, during which the influence of the ambient flow becomes negligible and the front location evolves as \(r^{2/3}\). At late times, the influence of inertial forces subsides and a new balance between gravitational and viscous forces forms, resulting in the front location advancing as \(r^{1/3}\). These scaling laws have been confirmed experimentally by a number of authors, among them [34–36].

3.1 Box Models. Conceptually simple box models, which can be formulated for both two-dimensional and axisymmetric geometries, usually do not consider ambient fluid entrainment. They assume that the gravity current evolves in the form of constant area rectangles, and neglect any vertical or streamwise variations inside the current. The box model concept was extended to turbidity currents by Dade and Huppert [37], and Gladstone and Woods [38], and it is reviewed in some detail by Ungarish [2].

3.2 Shallow Water Models. Gravity currents whose length is substantially larger than their depth can frequently be approximated by depth-averaged or shallow water models. This approach was introduced for compositional gravity currents by Rottman and Simpson [36], and later extended to turbidity currents [39–41]. Earlier reviews are provided in Refs. [14,15] and [17] and especially in the recent book by Ungarish [2]. In contrast to the above box models, shallow water models allow for streamwise variations in the current height and velocity. They consider the current to be well mixed, so that there are no density or velocity variations in the vertical direction. Furthermore, they assume that viscous forces are negligible and that vertical accelerations are small, so that the pressure field is purely hydrostatic. Entrainment at the top of the current is typically neglected, so that ambient fluid is usually neither entrained nor detrained, although Johnson and Hogg [42] recently introduced a shallow water model for gravity currents with entrainment. Kowalski and McElwaine [43] develop a shallow water model that accounts for variations in the lateral direction, which is frequently important in avalanche or granular flows. Other recent extensions model the effects of nonrectangular cross sections [44–46]. When employing shallow water models, the implications of the assumptions underlying these models need to be carefully considered on a case-by-case basis, since they may be too restrictive for some applications. For example, under some conditions turbulent mixing may result in substantial entrainment along the current-ambient interface. Similarly, during the later stages of turbidity currents, when the decaying turbulence is no longer fully able to distribute the particles across the entire current height, significant detrainment may occur.

For deeply submerged gravity and turbidity currents, the motion of the overlying fluid can frequently be neglected to a good approximation, which results in the so-called single-layer shallow water equations for the current height \(h(x, t)\) and velocity \(u(x, t)\) as a function of the streamwise coordinate \(x\) and time \(t\)

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = -g' \frac{\partial h}{\partial x} \tag{11}
\]

\[
\frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} = 0 \tag{12}
\]

These equations, which express the conservation of mass and momentum for the current fluid, represent a hyperbolic system in which the current front appears as a shocklike discontinuity. Hence, they cannot capture the detailed dynamics in the vicinity of the current tip and require a closure in terms of a relationship between the current velocity and its fractional height, which can be provided by models such as those in Refs. [19] and [24] discussed above. Under certain conditions, their relative simplicity allows for the derivation of similarity solutions that have been confirmed by numerous laboratory experiments, cf. the discussion by Linden [47]. Consequently, the shallow water equations have been successfully applied to a large variety of flow situations. One example is given in the work of Gonzalez-Juez and Meiburg [48], who extended earlier models by Lane-Serff et al. [49] and others in order to investigate the interaction of gravity currents with submarine pipelines. When compared to depth-resolving, high-resolution simulations, the shallow water estimates of the maximum drag were typically found to be accurate to within \(O (10\%)\). On the other hand, these models do not give information on such aspects as vortex shedding from the pipeline, along with the associated excitation frequencies. In another example, Birman et al. [50] employ a shallow water model for turbidity currents overflowing the levees of a submarine canyon. Their analysis demonstrates that the entrainment of ambient fluid by the overlying current governs the shape of the evolving levee. Negligible entrainment rates are seen to lead to exponentially decaying levee shapes, whereas power-law levee shapes emerge for constant entrainment rates.

For shallow gravity currents, i.e., when the depth of the overlying ambient fluid layer is of the same order as the current height, it is necessary to account for the dynamics of the ambient fluid and its coupling with the gravity current. This can be accomplished by formulating the so-called two-layer shallow water equations [11], which account for the conservation of mass and streamwise momentum in both the current and the ambient. Such two-layer models are reviewed in some detail in the recent book by Ungarish [2].

4 Depth-Resolving Simulation Approaches

In the following, we will begin by formulating the set of governing equations commonly employed for depth-resolved simulations. Subsequently, we will review the different simulation approaches based on these equations, such as DNS, LES, and RANS simulations. A discussion of some representative results obtained via these simulation approaches will follow.
4.1 Governing Equations. In many situations of interest, compositional gravity currents and turbidity currents are driven by small density differences not exceeding $O$ (1%). Under such conditions, the Boussinesq approximation can be employed, which treats the density as constant in the momentum equation with the exception of the body force terms. When dealing with turbidity currents, we account for the dispersed particle phase by means of an Eulerian–Eulerian formulation, which means that we employ a continuum equation for the particle concentration field, rather than tracking particles individually in a Lagrangian fashion.

In the following, it will be important to carefully distinguish between dimensional and dimensionless variables. Toward this end, we will employ the tilde symbol to indicate a dimensional variable, whereas variables without the tilde symbol are dimensionless. Under the Boussinesq approximation, the dimensional governing equations for compositional gravity currents driven by salinity and/or temperature gradients can be written as

$$\frac{\partial \bar{u}_i}{\partial t} = 0$$

(13)

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial (\bar{u}_i \bar{u}_j)}{\partial x_j} = -\frac{1}{\rho_1} \frac{\partial \bar{p}}{\partial x_i} + \bar{\nu} \frac{\partial^2 \bar{u}_i}{\partial x_k \partial x_k} + \frac{\bar{\rho} \bar{\rho}_1}{\rho_1} \mathbf{e}_i^p$$

(14)

$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial (\bar{\rho} \bar{u}_j)}{\partial x_j} = \bar{z} \frac{\partial^2 \bar{\rho}}{\partial x_i \partial x_i}$$

(15)

where $\bar{u}_i$ denotes the velocity vector, $\bar{\rho}$ the pressure, $\bar{\rho}$ the density, $g$ the gravitational acceleration, $\mathbf{e}_i^p$ the unit vector pointing in the direction of gravity, $\bar{\nu}$ the kinematic viscosity, and $\bar{z}$ the molecular diffusivity of the density field. We nondimensionalize Eqs. (13)–(15) by a reference length scale, such as the domain half-height $H/2$ of a lock-exchange flow (Fig. 3), the current density $\rho_1$, and the buoyancy velocity $\bar{u}_b$

$$\bar{u}_b = \sqrt{g H/2}$$

(16)

where $\sqrt{g}$ indicates the reduced gravity

$$\sqrt{g} = \frac{\sqrt{g}}{\bar{\rho}_1 - \bar{\rho}_2}$$

(17)

where $\bar{\rho}_2$ represents the ambient density. For partial depth lock releases, it may be more appropriate to form the relevant buoyancy velocity with the lock height, rather than the channel depth. After nondimensionalization, we obtain

$$\frac{\partial \tilde{u}_i}{\partial \tilde{t}} = 0$$

(18)

$$\frac{\partial \tilde{u}_i}{\partial \tilde{t}} + \frac{\partial (\tilde{u}_i \tilde{u}_j)}{\partial \tilde{x}_j} = -\frac{\partial \tilde{p}}{\partial \tilde{x}_i} + \frac{1}{\text{Re} \tilde{x}_k \partial \tilde{x}_k} + \tilde{\rho} \tilde{e}_i^p$$

(19)

$$\frac{\partial \tilde{\rho}}{\partial \tilde{t}} + \frac{\partial (\tilde{\rho} \tilde{u}_j)}{\partial \tilde{x}_j} = \frac{1}{\text{Re} \text{Sc} \tilde{x}_k \partial \tilde{x}_k}$$

(20)

Here the nondimensional pressure $\tilde{p}$ and density $\tilde{\rho}$ are given by

$$\tilde{p} = \frac{\bar{p}}{\bar{\rho}_1 \bar{u}_b^2}, \quad \tilde{\rho} = \frac{\bar{\rho} - \bar{\rho}_2}{\bar{\rho}_1 - \bar{\rho}_2}$$

(21)

The nondimensionalization of the governing equations gives rise to two dimensionless parameters in the form of the Reynolds number $\text{Re}$ and the Schmidt number $\text{Sc}$

$$\text{Re} = \frac{\bar{u}_b H}{2 \bar{\nu}}, \quad \text{Sc} = \frac{\bar{\nu}}{\bar{z}}$$

(22)

While the Reynolds number indicates the ratio of inertial to viscous forces, the Schmidt number represents the ratio of kinematic fluid viscosity to molecular diffusivity of the density field. We remark that the aspect ratio of the lock and, for partial depth lock releases, the ratio of the lock height to the channel height enter as an additional dimensionless parameter.

When the driving density difference is due to gradients in particulate loading, rather than salinity or temperature gradients, the above set of equations no longer provides a full description of the flow. Particles settle within the fluid, so that the scalar concentration field no longer moves with the fluid velocity. In addition, particle–particle interactions can result in such effects as hindered settling [51], increased effective viscosity and non-Newtonian dynamics [52], thereby further complicating the picture. However, away from the sediment bed turbidity currents are often quite dilute, with the volume fraction of the suspended sediment phase being well below $O$ (1%). Under such conditions, particle–particle interactions can usually be neglected, so that the particle settling velocity remains as the key difference (along with erosion) that distinguishes turbidity currents from compositional gravity currents. Due to the small particle volume fraction of dilute turbidity currents, the volumetric displacement of fluid by the particulate phase can usually be neglected, so that we can consider the fluid velocity field as divergence free. Rather, the particle–fluid interaction occurs primarily through the exchange of momentum, so that it suffices to account for the presence of the particles in the fluid momentum equation. In the following, we assume that the particle diameter $d_p$ is smaller than the smallest length scale of the flow, such as the Kolmogorov scale in turbulent flow. In addition, we consider only particles whose aerodynamic response time $t_p$ is significantly smaller than the smallest time scale of the flow $\tau'$, so that the particle Stokes number $St = t_p / \tau' \ll O(1)$ [53]. Here the aerodynamic response time is defined as

$$t_p = \frac{\bar{\rho}_p d_p^2}{18 \bar{\mu}}$$

(23)

with $\bar{\rho}_p$ indicating the particle material density and $\bar{\mu}$ denoting the dynamic viscosity of the fluid. Such particles can then be assumed to move with a velocity $\tilde{u}_p$, that is obtained by superimposing the local fluid velocity $\tilde{u}$ and the particle settling velocity $\tilde{u}_p \mathbf{e}_i^p$

$$\tilde{u}_p = \tilde{u}_i + \tilde{u}_i \mathbf{e}_i^p$$

(24)

where $\tilde{u}_i$ follows from balancing the gravitational force with the Stokes drag force

$$\tilde{F}_i = 5 \pi \bar{\mu} \bar{\rho}_p (\bar{u}_i - \tilde{u}_p)$$

(25)

as

$$\tilde{u}_i = \frac{d_i^2 (\bar{\rho} - \bar{\rho}_2) g}{18 \bar{\mu}}$$

(26)

Note that this implies that the particle velocity field is single-valued and divergence free, so that monodisperse particles do not, for example, accumulate near stagnation points or get ejected from vortex centers. Hence, we can describe the spatio-temporal evolution of the particle number concentration field $\zeta$ in an Eulerian fashion by the transport equation

$$\frac{\partial \zeta}{\partial \tilde{t}} + \frac{\partial \left( \zeta (\tilde{u}_i + \tilde{u}_i \mathbf{e}_i^p) \right)}{\partial \tilde{x}_i} = \bar{z} \frac{\partial^2 \zeta}{\partial \tilde{x}_i \partial \tilde{x}_i}$$

(27)

The diffusion term in Eq. (27) represents a model for the decay of concentration gradients due to the hydrodynamic...
diffusion of particles and/or slight variations in particle size and shape [51,54].

The motion of the fluid phase is described by the incompressible continuity equation and the Navier–Stokes equation augmented by the force exerted on the fluid by the particles, which is equal and opposite to the Stokes drag force acting on the particles. In dimensional form, these equations read

$$\frac{\partial \tilde{u}_j}{\partial \tilde{t}} = 0$$ \hspace{1cm} (28)

$$\frac{\partial \tilde{u}_i}{\partial \tilde{t}} + \tilde{\rho} \frac{\partial (\tilde{u}_i \tilde{u}_j)}{\partial \tilde{x}_j} = -\tilde{\rho} \frac{\partial \tilde{p}}{\partial \tilde{x}_i} + \tilde{\nu} \frac{\partial^2 \tilde{u}_i}{\partial \tilde{x}_j^2} + \frac{\tilde{c}_i}{\tilde{\rho}} \tilde{\epsilon}_i$$ \hspace{1cm} (29)

As we had done for compositional gravity currents, we use the domain half height $H/2$ and buoyancy velocity $u_b$ for nondimensionalization. The reduced gravity $\tilde{g}^+$ appearing in the calculation of $u_b$ can now be calculated as

$$\tilde{g}^+ = \frac{\pi (\tilde{\rho}_p - \tilde{\rho}) \tilde{v}_d \tilde{D}^3}{6\tilde{\rho}}$$ \hspace{1cm} (30)

where $\tilde{c}_0$ indicates a reference number concentration of particles in the suspension. After nondimensionalization, we obtain

$$\frac{\partial \tilde{u}_j}{\partial \tilde{t}} = 0$$ \hspace{1cm} (31)

$$\frac{\partial \tilde{u}_i}{\partial \tilde{t}} + \tilde{\rho} \frac{\partial (\tilde{u}_i \tilde{u}_j)}{\partial \tilde{x}_j} = -\tilde{\rho} \frac{\partial \tilde{p}}{\partial \tilde{x}_i} + \tilde{\nu} \frac{\partial^2 \tilde{u}_i}{\partial \tilde{x}_j^2} + \tilde{c}_i \tilde{\epsilon}_i$$ \hspace{1cm} (32)

$$\frac{\partial \tilde{c}}{\partial \tilde{t}} + \tilde{\epsilon}(\tilde{u}_i \tilde{u}_j) = \frac{1}{\tilde{\Re} \tilde{S}_{ij} \partial \tilde{x}_j}$$ \hspace{1cm} (33)

For polydisperse suspensions containing particles of different sizes, the above approach can easily be extended by solving one concentration equation for each particle size and corresponding settling velocity [55]. Note that the set of governing equations for turbidity currents (31)–(33) differs from the corresponding set for compositional gravity currents (18)–(20) only by the additional settling velocity term in the concentration equation. In the following, we employ Eqs. (31)–(33) for both types of currents, with the tacit assumption that the settling velocity vanishes for compositional gravity currents.

### 4.2 DNS

DNS represent the most accurate computational approach for studying gravity currents. In DNS all scales of motion, from the integral scales dictated by the boundary conditions down to the dissipative Kolmogorov scale determined by viscosity, are explicitly resolved. However, for the case of turbidity currents, when the particle diameter is smaller than the Kolmogorov scale, the fluid motion around each particle is usually not resolved, due to the prohibitive computational cost. Nevertheless, the drag law accurately captures the exchange of momentum between the two phases at scales smaller than the Kolmogorov scale, so that the approach described above is still referred to as DNS.

Consistent with the above arguments, the grid spacing required for DNS is of the order of the Kolmogorov scale, while the time step needs to be of the same order as the time scales of the smallest eddies. Because of the large disparity between integral and Kolmogorov scales at high Reynolds numbers, the computational cost of DNS scales as $Re^3$, so that the DNS approach is effectively limited to laboratory scale Reynolds numbers. The first DNS simulations of gravity currents in a lock-exchange configuration, such as the one shown in Fig. 3, were reported by Härtel et al. [56] for $Re = 1225$. Necker et al. [57] extended this work to turbidity currents at $Re = 2240$. More recent simulations of lock-exchange gravity currents by Cantero et al. [58] were able to reach $Re = 15,000$, which corresponds to a laboratory scale current of height 0.5 m with a front velocity of 3 cm/s.

DNS simulations can provide detailed information on the structure and statistics of the flow, on the various components of its energy budget, on the mixing behavior and many additional aspects. As a case in point, the simulations by Härtel et al. [56] explored the detailed flow topology near the current front and demonstrated that the stagnation point is located at a significant distance behind the nose of the current. DNS results are furthermore very useful for testing the accuracy and identifying any deficiencies in larger scale LES and RANS models [59]. Thus, while they are currently limited to laboratory scale currents, DNS simulations represent an excellent research tool for exploring the detailed physics of moderate Reynolds number gravity currents, and for constructing larger scale models for higher Reynolds number applications.

### 4.3 Large-Eddy Simulation

LES require fewer computational resources than DNS, as they resolve only the energy containing large eddies, while modeling all of the scales of motion below a cutoff. This approach often results in the majority of the dissipative scales being modeled [60,61]. The cutoff is determined by a filter width that depends on the grid spacing employed in the simulation. In the LES technique, the governing equations are obtained by filtering the Navier–Stokes and the concentration equations. The filtered governing equations read

$$\frac{\partial \tilde{u}_i}{\partial \tilde{t}} + \tilde{\rho} \frac{\partial (\tilde{u}_i \tilde{u}_j)}{\partial \tilde{x}_j} = -\tilde{\rho} \frac{\partial \tilde{p}}{\partial \tilde{x}_i} + \tilde{\nu} \frac{\partial^2 \tilde{u}_i}{\partial \tilde{x}_j^2} + \tilde{c}_i \tilde{\epsilon}_i$$ \hspace{1cm} (34)

$$\frac{\partial \tilde{c}}{\partial \tilde{t}} + \tilde{\epsilon}(\tilde{u}_i \tilde{u}_j) = \frac{1}{\tilde{\Re} \tilde{S}_{ij} \partial \tilde{x}_j}$$ \hspace{1cm} (35)

$$\frac{\partial \tilde{c}}{\partial \tilde{t}} + \tilde{\epsilon}(\tilde{u}_i \tilde{u}_j) = \frac{1}{\tilde{\Re} \tilde{S}_{ij} \partial \tilde{x}_j}$$ \hspace{1cm} (36)

Here $\tilde{u}_i$ and $\tilde{c}$ denote the filtered velocity and concentration fields, respectively. The effect of all scales of motion below the filter width appears as additional subgrid-scale (SGS) stress terms $\tau_{ij}$ in the momentum equation, and as an SGS flux term $\eta_j$ in the concentration equation. In LES implementations, the SGS terms $\tau_{ij}$ and $\eta_j$ are frequently modeled using an eddy viscosity approximation, which relates the SGS stresses to the resolved strain rate $\tilde{S}_{ij}$. The SGS stresses are thus calculated from

$$\tau_{ij} = \frac{1}{3} \tilde{\tau}_{ik} \delta_{ij} = -2\nu_t \tilde{S}_{ij}$$ \hspace{1cm} (37)

$$\tilde{S}_{ij} = \frac{1}{2} \left( \frac{\partial \tilde{u}_i}{\partial \tilde{x}_j} + \frac{\partial \tilde{u}_j}{\partial \tilde{x}_i} \right)$$ \hspace{1cm} (38)

where $\nu_t$ denotes the SGS eddy viscosity, which is calculated using the Smagorinsky model [62] as

$$\nu_t = (C_s \Delta)^2 |\tilde{S}|$$ \hspace{1cm} (39)

where $|\tilde{S}|$ indicates the magnitude of the resolved strain rate tensor, $\Delta$ represents a length scale proportional to the local grid spacing, and $C_s$ denotes an empirical model coefficient that needs to be specified

$$|\tilde{S}| = \sqrt{2 \tilde{S}_{ij} \tilde{S}_{ij}}$$ \hspace{1cm} (40)

$$\Delta = \sqrt{(\Delta_x \Delta_y \Delta_z)}$$ \hspace{1cm} (41)

In the above, $\Delta_x$, $\Delta_y$, and $\Delta_z$ represent the local grid spacings along the $x$-, $y$-, and $z$-directions, respectively. Analogous to the SGS
stress terms, the SGS flux terms are calculated using an eddy diffusivity model as

$$\eta_j = - \frac{\nu_t}{Sc_t} \frac{\partial \bar{c}}{\partial x_j}$$  \hspace{1cm} (42)$$

Here $Sc_t$ denotes the SGS Schmidt number. Both the model coefficient $C_t$ and the SGS Schmidt number $Sc_t$ need to be specified either a priori, or calculated during the simulation. Usually more accurate LES predictions can be achieved by employing the so-called dynamic procedure [63,64], which samples the resolved flow field to estimate the model coefficient $C_t$ and the SGS Schmidt number $Sc_t$. Such dynamic models usually yield higher accuracy by adjusting the dissipation provided by the SGS model to local flow conditions, allowing it to vanish in laminar regions.

Similar to DNS, LES also yields detailed, time-dependent large-scale flow structures. In fact, flow statistics that depend mostly on the large-scale structures such as mean velocity and concentration, turbulence intensities, scalar fluctuations, bed shear stress, etc., can be estimated reasonably accurately on the basis of LES simulations. Stevens et al. [65] demonstrate that LES, at sufficiently high resolutions, can reproduce accurately higher order statistics such as even-order moments of the streamwise velocity fluctuations up to order 10. Because of the much lower computational cost, LES simulations have been conducted for significantly higher Reynolds numbers than DNS. Ooi et al. [66] carried out LES simulations for gravity currents at Reynolds numbers of $O(10^5 - 10^6)$. These simulations show that the Kelvin–Helmholtz billows become three-dimensional sooner than for moderate Reynolds number gravity currents, and that the dissipation rate becomes independent of time. For field scale flows, the Reynolds number can become as large as $O(10^6 - 10^7)$. Though LES can be applied at any Reynolds number for free shear flows, the presence of a solid bottom boundary in gravity and turbidity currents limits LES to Reynolds numbers lower than those field scale values, because of the need to resolve the smaller but energy containing eddies in the inner layer, near-wall region. There exists the possibility of applying LES to field scale flows without resolving the inner layer, by representing this layer in an average sense through a wall-layer model. The limitations and drawbacks of various such wall-layer models are discussed by Piomelli and Balaras [67]. For attached geophysical flows over both smooth and rough boundaries, Radhakrishnan and Piomelli [68] demonstrate that equilibrium stress based wall-layer models can predict the flow evolution accurately.

4.4 RANS Simulations. This approach numerically integrates the RANS equations, which are obtained by ensemble-or-time-averaging the governing continuity, momentum, and concentration equations [69]. Unlike eddy-resolving DNS or LES methods, RANS simulations only yield the mean flow field. The governing RANS equations are structured similarly to the LES equations (35) and (36), except that the bar symbol now refers to the ensemble-averaged flow field rather than the filtered field, and the additional terms $\frac{\partial \tau_{ij}}{\partial x_j}$ and $\frac{\partial \eta_j}{\partial x_j}$ now represent the Reynolds stresses and the scalar fluxes, respectively. Additional closure equations for computing the Reynolds stresses and scalar fluxes are discussed by Wilcox [69]. The most common RANS closure is based on the solution of a transport equation for the turbulent kinetic energy $k$ and the dissipation rate $\varepsilon$. In the context of turbidity currents, Choi and Garcia [70] applied the $k-\varepsilon$ model to flows propagating down a slope. They employ the boundary layer approximation of the governing equations to reduce the computational cost, and they report the impact of the model constants on the predicted rate of water entrainment. More recently, Sequeiros et al. [71] utilized the $k-\varepsilon$ model along with an Exner equation for the sediment transport [72,73], to study buoyancy-reversing turbidity currents and the associated bed form evolution. Abd El-Gawad et al. [74] invoked the Mellor–Yamada turbulence closure to study the evolution of turbidity currents propagating over complex topography in a submarine environment that includes canyons, fans, and sinuous channels in the Niger delta. The authors vary the inflow conditions to match the sediment data obtained at seven piston cores, and they demonstrate that the RANS approach can be utilized to compute the inflow conditions from the core data. The simulation results allow for the prediction of the resulting sediment deposit patterns.

While the RANS approach is computationally less expensive than LES or DNS and thus allows us to carry out simulations for km-scale currents, its predictions generally are also less accurate. As Choi and Garcia [70] show, it may be necessary to tune the model constants to local flow conditions in order to improve the quality of RANS predictions. These difficulties reflect the fact that the Reynolds stresses appearing in the RANS equations depend on the large flow structures, which in turn vary with the boundary and flow conditions. Hence, it is challenging to construct a universal RANS model. In contrast, the SGS terms appearing in the LES equations depend mainly on the smaller, more universal unresolved eddies which are independent of the local flow conditions.

4.5 Gravity and Turbidity Currents Propagating Over Flat Terrain. The lock-exchange setup sketched in Fig. 3 represents the most popular configuration for conducting laboratory experiments on gravity and turbidity currents. Furthermore, its geometrical simplicity and straightforward initial and boundary conditions render it ideal for depth-resolving, two- and three-dimensional simulations as well. The first DNS simulation of compositional gravity currents were reported by Härtel et al. [56] for a Reynolds number of 1225. In two dimensions, these authors employed a vorticity-streamfunction formulation in conjunction with a mixed spectral/compact finite difference discretization. Their three-dimensional simulations, on the other hand, relied on primitive variables, along with a mixed spatial discretization based on a spectral-element collocation technique in the vertical direction and Fourier expansions in the stream- and spanwise directions. The temporal discretization was semi-implicit.

Figure 4 displays the temporal evolution of the flow field by means of three-dimensional density isosurfaces, with additional density contours shown in the side plane. Beyond $t = 10$, the well-known lobe-and-cleft structure of the front can be clearly recognized. This structure evolves from a spanwise instability of the front whose linear growth was analyzed in detail by Härtel et al. [75]. The shear along the interface between the two fluids gives rise to pronounced Kelvin–Helmholtz vortices. Initially, these vortices are predominantly two-dimensional and extend across the whole channel span. Beyond $t = 15$, they lose their spanwise coherence and acquire a strongly three-dimensional structure. These simulations provide detailed insight into the physics of the current, and in particular into the evolution of the front. Up until then, it had been assumed that the nose of the current coincided with a stagnation point, in a coordinate system moving with the current front. However, the numerical simulations revealed that the stagnation point is located below and behind the nose of the current.

Necker et al. [57] extended these simulations to turbidity currents and conducted a detailed comparison between compositional and particle-laden currents. Their simulations were performed at a significantly higher Reynolds number of 2240. These authors found the initial evolution of gravity and turbidity currents to be quite similar, with lobe-and-cleft structures at the front and Kelvin–Helmholtz billows along the interface. During the later stages, however, turbidity currents were seen to lose their potential energy more rapidly, due to the settling of the particles. This causes turbidity currents to slow down earlier as compared to gravity currents. The rate at which a turbidity current loses particles initially grows rapidly, as the result of its increasing length. Later on, as the particle concentration of the current decreases, the sedimentation rate decreases along with it. Figure 5 compares
deposit profiles from a two-dimensional simulation with corresponding experimental data of de Rooij and Dalziel [76]. Here, the deposit profile is computed as a function of time as

\[ D_t(x_1, t) = \frac{1}{L_s} \int_0^{L_s} c_w(x_1, \tau) u_s d\tau \]  

(43)

where \( c_w \) is the particle concentration value at the wall. As the figure illustrates, the numerically evaluated deposit profile agrees well with the experimental data downstream of the lock region. The differences between the numerical profile and the experimental data in the lock region are likely due to the initial stirring of the suspension in the experiments which is not represented in the simulation, as well as to particle sedimentation before the start of the experiment.

Fig. 4  Full-depth, lock-exchange Boussinesq gravity current at Re = 1225. The flow is visualized at different times \( t \) by means of the three-dimensional density isosurface \( \rho = 0.5 \), along with density contours in the side-plane. Initial parameters: \( L = 23, l = 10, H = 2, d = 2, \) and \( W = 3 \) (Reprinted with permission from Hartel et al. [56]. Copyright 2000 by Cambridge University).
In a subsequent investigation, Necker et al. [77] compared the energy budgets of gravity and turbidity currents, cf. Fig. 6. The figure shows the time history of potential energy ($E_p$), kinetic energy ($k$), viscous dissipation due to the resolved motion ($E_d$), and viscous dissipation in the unresolved small-scale Stokes flow around the individual particles ($E_s$). All energy components are normalized by the initial potential energy $E_{p0}$. Figure 7 illustrates the development of a turbidity current via isosurfaces of particle concentration at various instants in time. More than half of the potential energy is converted into kinetic energy during the first 2–3 units of time (cf. Fig. 7(b)) when the fluid in the lock region starts a convective motion from rest. As the length of the current increases with time (Figs. 7(c) and 7(d)), the interfacial area with strong velocity gradients increases, which results in increasing viscous dissipation $E_d$ with time. For the specific parameter values investigated in Ref. [77], the authors find that, by the time the current comes to rest, as much as 40% of the initial potential energy has been lost in the unresolved Stokes flow around the particles. The spatial distribution of the viscous dissipation shows two peak regions, one in the boundary layer close to the wall and one near the interface between the two fluids. Close to 15% of the total dissipation occurs in the thin boundary layer region near the wall.

Necker et al. [77] furthermore employed Lagrangian markers to study the mixing between interstitial and ambient fluid, both of which have the same density. Their analysis showed the perhaps somewhat counterintuitive result that the two fluids become most thoroughly mixed for intermediate particle settling velocities. For very small settling velocities, the particles remain suspended in the interstitial fluid for very long times, dragging it toward the bottom wall and preventing it from mixing. For very large settling velocities, on the other hand, the particles settle out before much of their potential energy can be converted into kinetic energy, which again prevents strong mixing. For intermediate settling velocities, the particles remain suspended sufficiently long so that much of their potential energy is converted into kinetic energy and a vigorous flow develops, but they settle out sufficiently fast so that clear, neutrally buoyant interstitial fluid forms that has sufficient kinetic energy left to mix with the ambient fluid.

While most lock-exchange simulations assume the lock fluid to be at rest initially, this is usually not the case in corresponding experiments. First of all, the removal of the partition injects a certain amount of energy into the fluid. Second, for lock-exchange experiments involving turbidity currents the suspension usually has to be stirred vigorously right before the removal of the partition, to ensure that the particles are well mixed across the fluid column. In order to analyze the effects of initially present kinetic energy on the flow evolution, Necker et al. [77] performed additional simulations in which the lock fluid initially contained 12.5% and 25% of the initial potential energy as kinetic energy. While this initial kinetic energy dissipates relatively quickly, it nevertheless significantly enhances the mixing within the current and causes it to travel slightly faster during the late stages of its evolution. More recently, Espath et al. [78] explore the dependence of key flow features on $Re$ by conducting DNS simulations of lock-exchange turbidity currents up to $Re = 10^4$.

The lock aspect ratio $\lambda$, which is defined as the ratio of the lock length $x_0$ to the lock height $h_0$, plays an important role in the flow development. Bonometti et al. [79] performed two-dimensional Navier–Stokes simulations to investigate the effect of the lock aspect ratio on the shape and motion of the current. For a current with an initial density difference of 1%, Fig. 8 shows the influence of $\lambda$ on the shape of the current. Within three units of lengths of the front, the shape of the current is seen to depend only weakly on $\lambda$. Farther away from the front, however, the height of the current body depends strongly on $\lambda$, as it is larger than the head height for $\lambda \geq 6.25$, and smaller for $\lambda \leq 1$. The current speed during the slumping phase is largely independent of $\lambda$ for $\lambda \geq 6.25$. However, the current speed during the slumping phase decreases with $\lambda$ for $\lambda \leq 1$.

Cantero et al. [80] study the front velocity of the gravity current during the acceleration, slumping, inertial, and viscous phases by means of two- and three-dimensional simulations at various

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**Figure 5** Nondimensional particle deposit profiles as function of the streamwise coordinate. Results are shown for times $t = 7.3$ and $t = 10.95$, along with the final profile ($t \to \infty$) after all particles have settled out. Solid line: two-dimensional simulation, dashed line: experimental data of de Rooij and Datziel [76]. In both cases $Re = 10,000$, $u_s = 0.02$, $l = 0.75$, and $H = d = 2.0$ (Reprinted with permission from Necker et al. [57]. Copyright 2002 by Elsevier).

**Figure 6** (a) Time history of potential energy $E_p$ and kinetic energy $k$. (b) Time history of the dissipation $E_d$ due to the resolved motion, and the dissipation $E_s$ due to the unresolved Stokes flow around the individual particles. All energy components are normalized by the initial potential energy $E_{p0}$. Solid (dashed) lines indicate turbidity (gravity) current results. The dotted-dashed line in (a) gives the sum of $E_p$, $k$, $E_d$, and $E_s$ for the turbidity current, $Re = 2240$. Initial parameters: $L = 23$, $I = 1$, $H = 2$, $d = 2$, and $W = 2$ (Reprinted with permission from Necker et al. [77]. Copyright 2005 by Cambridge University).
Reynolds numbers. They compare the numerical front velocity predictions with corresponding experimental data and with theoretical predictions. Figure 9 shows the temporal variation of the front velocity for three different Reynolds numbers. During the constant velocity phase, the lower Reynolds number currents travel more slowly than their higher Reynolds number counterparts. For all currents, the front velocity begins to depart from the constant velocity phase around \( t \approx 12 \). For the two lower Reynolds number currents, the subsequent decrease of the front velocity shows good agreement with the theoretical predictions for the viscous phase. For the high Reynolds number current, after the constant velocity phase the front velocity agrees well with the inertial phase scaling by Huppert and Simpson [34].

Cantero et al. [58] performed DNS simulations of planar gravity currents at Reynolds numbers of 8950 and 15,000, respectively, for which the flow is fully turbulent. They find that the turbulence in the near-wall region shows remarkable resemblance to the turbulence in standard wall-bounded flows. Specifically, this region exhibits several hairpin vortices, as well as low- and high-speed streaks similar to the ones observed in classical boundary layers. Near the current front, the low- and high-speed streaks are about 200 wall units apart. Farther behind the current front, where the turbulence in the near-wall region becomes fully developed, the spacing between the streaks reduces to about 100 wall units. Here a wall unit \( x^+ \) is defined as 
\[
x^+ = x \cdot \frac{u_\text{ref} h}{v},
\]
where \( u_\text{ref} = \sqrt{\left(\tau_{\text{wall}}/\rho\right)} \) represents the friction velocity and \( \tau_{\text{wall}} \) denotes the shear stress at the wall. In the interface region, Kelvin–Helmholtz vortices are generated near the front due to shear. These vortices subsequently break up and generate smaller scale turbulence behind the current front.

Ooi et al. [66] employ LES simulations to study compositional gravity currents at Reynolds numbers ranging from 3000 to 10^6, where the flow is again fully turbulent. Figure 10 depicts the flow structure in the near-wall region for \( Re = 87,750 \). In particular, Fig. 10(b) shows streamwise velocity contours in a plane located about 11 wall units from the bottom wall. High- and low-speed streaks in the near-wall region are clearly visible, similar to those observed in turbulent boundary layers. The average width of these streaks is approximately 40 wall units, and their average length decays from approximately 1200 wall units near the front to about 800 wall units toward the end of the streaky region. Although the near-wall flow structure resembles that of a turbulent boundary layer, the streaks here are larger in terms of wall unit dimensions as compared to those in traditional turbulent boundary layers. This discrepancy can be attributed to the different flow conditions in gravity currents, such as the presence of the mixing layer at the interface. Constantinou [82] provides a summary of recent LES simulations for lock-exchange gravity currents.

### 4.6 Turbidity Currents Interacting With Seafloor Topography

With regard to gravity and turbidity currents interacting with complex topography, one line of research addresses the changes in the current dynamics brought about by the topography. A host of new flow phenomena can arise as a result of such interactions, including hydraulic jumps, reflected solitary waves, and strong localized vortical structures. Equally interesting, however, is the question as to how the seafloor topography is altered by the turbidity currents via erosion and deposition, resulting in the formation of sediment waves, gullies, channels, levees, and fans. With regard to the latter, linear stability investigations can provide fundamental insight into the mechanisms by which such features can appear on an initially flat seafloor.

Traditionally, most such analyses have been based on depth-averaged equations, such as the cyclic-step theory [83–85]. More recently, however, depth-resolving linear stability investigations have provided additional insight into the coupling between the detailed flow structure inside the current and the sediment bed below. The investigation by Hall et al. [86] into the formation of channels and gullies by turbidity currents represents an instructive example in this regard. The stability analysis indicates that for such gullies to occur via a linear instability, above the sediment bed the suspended sediment concentration needs to decay more slowly than the streamwise velocity. Under such conditions, an upward protrusion of the sediment bed will find itself in an environment where erosion decays more quickly than sedimentation, and so it will keep increasing. Conversely, a local valley in the sediment bed will see erosion increase more strongly than sedimentation, which again will amplify the initial perturbation.

This base flow effect is modulated by the perturbation of the suspended sediment concentration and by the shear stress due to a secondary flow structure in the form of counter-rotating streamwise vortices, cf. the eigenfunctions shown in Fig. 11. The figure shows the shape of the sediment bed surface, along with the sediment concentration disturbance and the streamwise and transverse perturbation velocity fields, in a transverse plane. The shape of the sediment bed perturbation is shown in the bottom frame. The semicircular lines close to the sediment bed in the top and middle frames represent contours of the sediment concentration perturbation, with solid lines indicating positive values and dashed lines negative values. Streamlines of the transverse velocity perturbation are superimposed, with arrows denoting the direction of the flow. In the top frame, gray shading reflects the perturbation streamwise velocity, with lighter areas indicating positive values and darker areas negative values.
Above the peaks of the perturbed sediment bed, we observe a negative sediment concentration perturbation (reduced sediment loading), which results in lower hydrostatic pressure as compared to the troughs of the sediment bed, where the sediment concentration increases. Hence, a spanwise pressure gradient exists along the sediment bed surface, which drives a perturbation flow from the troughs to the peaks. Via the continuity equation, this spanwise perturbation flow along the sediment bed surface leads to the formation of the counter-rotating streamwise vortices visible in the figure. Note that above the sediment bed peaks, these vortices carry low-speed fluid, i.e., fluid with a small streamwise velocity, away from the sediment bed, while high-speed fluid from the free stream is brought toward the sediment bed at the troughs. In this way, the shear stress at the bed surface, which primarily is a function of the local streamwise velocity gradient, is enhanced above the troughs and lowered above the peaks. This is reflected by the middle frame of the figure, which shows the perturbation shear through gray shading, with lighter areas indicating positive values and darker areas negative values. Thus, erosion increases in the valleys and decreases at the peaks, which further amplifies the initial sediment bed perturbation.

In a similar vein, Lesshafft et al. [87] address the formation of deep-water sediment waves, via the interaction of an erodible sediment bed with a turbidity current. Their stability analysis demonstrates the existence of both Tollmien–Schlichting and internal wave modes in the stratified boundary layer. For the internal wave mode, the stratified boundary layer acts as a wave duct, whose height can be determined analytically from the Brunt–Väisälä frequency criterion. Consistent with this criterion, distinct unstable perturbation wave number regimes exist for the internal wave mode. For representative turbidity current parameters, the analysis predicts unstable wavelengths that are consistent with field observations. Furthermore, for most of the unstable wave number ranges, the phase relations between the sediment bed deformation and the associated wall shear stress and concentration perturbations are such that the sediment waves migrate in the upstream direction, which again is consistent with field observations.

While linear stability analysis represents a powerful tool for obtaining insight into the initial formation process of seafloor topography, the effects of such topography on gravity and turbidity currents can best be explored via laboratory experiments [88–92] and/or numerical simulations. Along these lines, Nasr-Azadani and Meiburg [55,93] employ DNS simulations to study the interaction of turbidity currents with a local seamount, cf. Fig. 12. Their representation of the seafloor topography is based on an...

Fig. 8 “Instantaneous shape of Boussinesq density currents \( (\rho_l/\rho_s = 1.01) \) at time \( t = 7 \). Isolines of \( (\rho_l - \rho_s)/(\rho_l + \rho_s) = 0.05, 0.25, 0.5, 0.75, \) and 0.95 are shown.” Initial parameters: \( l = x_0 \), \( H = 10 \), \( d = h_0 = 1 \), \( L = 37.5 \) for \( \lambda = 18.75 \) and \( L = 12.5 \) for all other \( \lambda \) values (Reprinted with permission from Bonometti et al. [79]. Copyright 2011 by Cambridge University).

Fig. 9 “Time evolution of the front velocity for planar currents from three-dimensional simulations. The plot also includes experimental data from two of the lower Reynolds number experiments by Marino et al. [81] with \( l = x_0 = x_0 = l = 1 \). Also included are the theoretical predictions for all phases of spreading. The viscous phase predictions are for \( Re = 8950, x_0 = 1 \) and \( h_0 = 1 \)” (Reprinted with permission from Cantero et al. [80]. Copyright 2007 by Cambridge University).
immersed boundary method [94–98]. Rather than terrain-following coordinates, this approach uses a Cartesian grid everywhere and modifies the fluid velocity and concentration at the nodes near the bottom boundary in order to satisfy the no-slip condition along with the appropriate boundary condition for the concentration field.

The authors explore the effects of the seamount’s height on the dynamics and depositional behavior of bidisperse turbidity currents. The seamount is seen to deflect the current laterally, which enhances the deposition toward the sides of the seamount while reducing it on the seamount itself. The strength of these effects varies with the height of the seamount, as well as with the grain size. Especially the tall seamount tends to transform the spanwise boundary layer vorticity into strong horseshoe vortices, while it also deforms the mixing layer vorticity into inverted horseshoe vortices. In this way, a complex vortical flow structure is generated in the neighborhood of the seamount, which in turn results in complicated erosion and deposition patterns. An alternative approach for representing complex seafloor topography involves the use of finite element methods. This line of research is pursued by Coutinho and colleagues [99–102], who apply parallel stabilized finite element methods with adaptive meshes to lock-exchange type gravity and turbidity currents. A spectral element approach toward simulating the propagation of gravity currents over complex seafloor topography is employed by Özgökmen et al. [103].

Most computational investigations to date are limited to nonerodible seafloors, although a few simulations are beginning to account for the erosion of sediment. Along these lines, Blanchette et al. [104] investigate two-dimensional turbidity currents propagating down an erodible inclined plane. They focused on the avalanche-like growth and acceleration of turbidity currents via the erosion and entrainment of particles from the sediment bed, which is simulated based on the empirical erosion model of Garcia and Parker [105]. Blanchette et al. [104] analyze the influence of such quantities as bottom slope angle, particle size, and lock height on the growth or decay of turbidity currents, with the goal of identifying critical thresholds beyond which such currents become self-sustaining. We note that the erosion model they employ is based on an idealized set of conditions, and that additional research into the dynamics of erosion and resuspension is required to obtain improved quantitative predictions. In a similar vein, the simulations reported by Strauss and Glinsky [106] and Hoffmann et al. [107] explore the nonlinear stages of sediment formation by turbidity currents.

4.7 Gravity Currents Interacting With Engineering Infrastructure. As briefly mentioned in the introduction, gravity and turbidity currents can pose considerable hazards to

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**Fig. 10** "Visualization of the flow structure in the near-wall region of a turbulent gravity current for Re = 87,750. (a) Vertical vorticity contours near the bottom wall; (b) streamwise velocity contours showing the high- and low-speed streaks in a plane located at about 11 wall units from the bottom wall. The light and dark vorticity contours in (a) correspond to $\omega_y = 2u_\nu/h$ and $\omega_y = -2u_\nu/h$, respectively" (Reprinted with permission from Ooi et al. [66]. Copyright 2009 by Cambridge University).

**Fig. 11** Dominant unstable eigenfunction modes for a turbidity current propagating over a plain erodible sediment bed. Shown is a plane normal to the main flow direction. "The solid and dashed lines depict positive and negative concentration perturbation contours, respectively. Streamlines of the transverse perturbation velocity field are superimposed, with arrows denoting the flow direction. In the top frame, gray shading reflects the perturbation $u$-velocity, with lighter areas indicating positive values, and darker areas negative values. The shape of the interface perturbation is shown in the bottom frame" (Reprinted with permission from Hall et al. [86]. Copyright 2008 by Cambridge University).
engineering infrastructure, such as submarine oil and gas pipelines, wellheads and telecommunication cables on the seafloor. Mitigation of these hazards requires detailed knowledge not only of the forces exerted by the currents but also of the associated frequencies, so that potential resonant vibrations of cables and pipelines can be avoided. Gravity currents can furthermore endanger submarine infrastructure via scour, which exposes and weakens these structures by removing the sediment around them. Complementing earlier laboratory experiments \cite{108,109}, computational investigations in recent years have matured to the point where they can add substantially to our insight into the interaction between gravity currents and obstacles of various shapes.

Gonzalez-Juez et al. \cite{110} employ two- and three-dimensional LES simulations to investigate the forces acting on a bottom-mounted square cylinder during the interaction with a lock-exchange gravity current, cf. Figs. 13–15. Their numerical approach is based on the nondissipative finite volume DNS/LES code of Pierce and Moin \cite{111}, which uses the dynamic Smagorinsky model to calculate the SGS stress terms. Utilizing a cartesian grid, the simulations represent the obstacle via a grid blanking methodology that sets the velocity, pressure, and concentration inside the obstacle to zero. Consistent with the experiments of Ermanyuk and Gavrilov \cite{108,109}, Gonzalez-Juez et al. observe impact, transient, and quasisteady stages during the current/obstacle interaction. During the impact stage, as the current approaches the obstacle, the drag force on the obstacle increases exponentially and reaches an overall maximum value. During the subsequent transient stage the drag fluctuates in time, while the final quasisteady stage sees a decrease in both the average drag and its temporal fluctuations.

Figure 13 shows a two-dimensional simulation for $Re = 2000$. The domain height $H$ is five times larger than the lock height $h$, and the obstacle is placed three lock heights downstream of the gate. For different times, the figure displays the concentration field along with instantaneous streamlines on the left, while the vorticity is shown on the right. During the impact stage, a small
separation region forms above and behind the obstacle. The transient stage starts as the current is deflected upward, which increases the size of the separation region. Some distance downstream of the obstacle the current plunges downward and reattaches to the bottom wall, which results in the temporary trapping of lighter ambient fluid in the wake behind the obstacle. Subsequently, this trapped lighter fluid is gradually flushed out of the obstacle’s wake by the denser fluid, and the quasisteady stage begins. The drag oscillation during the transient stage is caused primarily by this time-dependent nature of the separation region.

The computational simulation results provide us with the opportunity to extract detailed information about the flow field that might be more difficult to evaluate experimentally, such as the individual contributions to the various forces. Along these lines, Gonzalez-Juez et al. [110] examine the temporally varying drag and lift in Fig. 14, which shows the overall drag along with its individual components due to the pressure force on the upstream ($F_w$) and downstream ($F_e$) faces of the cylinder, and the viscous drag force ($F_v$) acting on the top surface of the cylinder. The sign of the pressure force is positive when it is directed from the fluid toward the solid wall. The overall drag is obtained as $F_D = F_w - F_e + F_v$.

The figure indicates that the overall drag is dominated by the pressure contributions, whereas the viscous contribution is negligible. In this graph, the impact, transient, and quasisteady stages can be associated with the time intervals $1 < t/(h/V) < 3.3$.

Fig. 13 Gravity current interacting with a bottom-mounted square cylinder. Temporal evolution of the concentration (left) and vorticity (right) fields. “Instantaneous streamlines in the laboratory reference frame are superimposed onto the concentration fields.” Initial parameters: $L = 24$, $I = 9$, $H = d = 1$ (Reprinted with permission from Gonzalez-Juez et al. [110]. Copyright 2009 by Cambridge University).
which causes a local pressure decrease. Subsequently, the lift indicates the presence of a recirculation zone above the cylinder, attains a maximum at the same time as the drag, when Fig. 13 increases rapidly as the current arrives at the cylinder. The lift ral variation of the lift. While the lift is negative initially, it resulting in the increase of the overall drag. Beyond pressure increase (decrease) on the upstream (downstream) side, change in flow structure around the cylinder is associated with a passing of a Kelvin–Helmholtz billow above the cylinder. This into the downstream recirculation zone, triggered perhaps by the $3 < t/(h/V) < 8.8$ and $t/(h/V) > 8.8$, respectively, where $V$ denotes the front velocity. The drag increase during the impact stage can be attributed to the increase in pressure on the upstream side of the cylinder as the gravity current approaches it. Subsequently, the drag decreases during the time interval $t/(h/V) \approx 3.3 - 4.4$, due to a decrease in $F_w$ and a simultaneous increase in $F_r$. This decrease in $F_w$ is due to the formation of a recirculation region upstream of the cylinder, as indicated by time frame $t/(h/V) = 4.1$ in Fig. 13. Concurrently, the recirculation zone behind the cylinder is convected downstream and away from the cylinder, so that $F_r$ increases.

During the time interval $t/(h/V) \approx 4.4 - 5.3$, the drag increases and reaches a second maximum. The vorticity field at $t/(h/V) = 4.7$ in Fig. 13 indicates that the clockwise vortex upstream of the cylinder has been convected past the cylinder and into the downstream recirculation zone, triggered perhaps by the passing of a Kelvin–Helmholtz billow above the cylinder. This change in flow structure around the cylinder is associated with a pressure increase (decrease) on the upstream (downstream) side, resulting in the increase of the overall drag. Beyond $t/(h/V) \approx 11$, the drag decreases again.

Figure 14(b) provides corresponding information for the temporal variation of the lift. While the lift is negative initially, it increases rapidly as the current arrives at the cylinder. The lift attains a maximum at the same time as the drag, when Fig. 13 indicates the presence of a recirculation zone above the cylinder, which causes a local pressure decrease. Subsequently, the lift decreases as this recirculation zone is convected downstream, and dense fluid sweeps over the cylinder during the interval $3 < t/(h/V) < 4.1$. A renewed increase in the lift is observed as vorticity is convected over the cylinder from $4.1 < t/(h/V) < 5$, which is followed by yet another decrease from $5 < t/(h/V) < 6.2$, when this vorticity enters the downstream recirculation region. During the late stages the lift remains negative, as dense fluid covers the cylinder. Just as for the drag, the figure demonstrates that the viscous contribution to the lift is negligible.

Gonzalez-Juez et al. [110] furthermore compare two- and three-dimensional LES simulations to the experimental data of Ermanyuk and Gavrilov [108] for the same geometric configuration, cf. Fig. 15. This comparison indicates that, while two-dimensional simulations accurately describe the impact stage, they overpredict the force fluctuations during the transient and quasisteady stages. This is a consequence of the three-dimensional nature of the Kelvin–Helmholtz vortices and the vortex shedding process in the experiments, so that it takes a three-dimensional simulation to reproduce this behavior.

For a circular cylinder mounted above the ground, Gonzalez-Juez et al. [112] investigate the effect of the gap size on the forces generated during the interaction with a gravity current. They find the gap size to have a negligible effect on the maximum drag during the impact stage, while the lift fluctuations increase with the gap size. During the quasisteady stage, the case of a gravity current flowing past a circular cylinder exhibits similarities with a corresponding constant density boundary layer flow past a
cylinder. For both flows, there is a critical gap size above which the forces begin to fluctuate due to vortex shedding from the cylinder. For small gap sizes, the interaction of the wake vorticity with the boundary layer vorticity can lead to the suppression of vortex shedding. Furthermore, in contrast to the constant density boundary layer flow, a cylinder interacting with a gravity current experiences a result of buoyancy, and also due to the deflection of the wake toward the wall.

Gonzalez-Juez et al. [113] explore the wall shear stress in the neighborhood of a circular cylinder mounted above a wall, to obtain insight into the nature of the scour that could be triggered at various stages of the interaction between the gravity current and the cylinder. Figure 16 shows the spanwise vorticity in the midplane at different times, for a three-dimensional simulation with Re = 9000, H/h = 2.5, D/h = 0.1, and G/h = 0.03. Here, H indicates the domain height, h denotes the lock height, D represents the diameter of the circular cylinder, and G is the gap size. Figures 16(b) and 16(c) show the formation of a jet of dense fluid in the gap during the impact stage. This jet continues during the transient stage (Fig. 16(d)), when the main current plunges downstream of the cylinder. These two processes are associated with high levels of shear stress at the bottom wall. The maximum friction velocity during the impact stage is about 60% larger than the quasisteady stage value, which suggests that scouring can be severe during the impact stage. There exists considerable spanwise variation in the friction velocity as a result of the lobe-and-cleft instability, with larger friction velocities observed near the lobes, rendering these the prime candidates for strong scour.

The above investigations are extended to arrays of cylindrical obstacles, as well as to obstacles in the form of dunes or sediment waves, by Tokyay et al. [114–116].

4.8 Stratification Effects. Density stratification can affect the dynamics of gravity and turbidity currents in different ways. In the context of atmospheric or oceanic flows, a gravity current may propagate within a stratified ambient fluid. This scenario was explored by Maxworthy et al. [10] via lock-exchange experiments and corresponding two-dimensional DNS simulations. In their setup, the ambient fluid was linearly stratified, so that it could give rise to internal waves with an intrinsic frequency and propagation velocity. Depending on whether the front velocity of the current is larger or smaller than this wave propagation velocity, one can distinguish between super- and subcritical currents. For subcritical currents, the authors found that internal wave interactions with the current resulted in an oscillatory front velocity. For supercritical currents, on the other hand, the front velocity was seen to decay monotonically. A corresponding one-layer shallow water model was developed by Ungarish and Huppert [117], although such models by their very nature cannot incorporate the effects of the internal waves in the ambient fluid. Detailed theoretical models for the related scenario of a gravity current propagating into a two-layer stratified ambient are developed by Flynn et al. [29] as well as by White and Helfrich [118]. In Ref. [119], the latter authors consider arbitrary density stratifications.

The effects of density stratification may be important not just in the ambient but also within the current itself. Again, box models and shallow water models for such stratified currents were developed by Zemach and Ungarish [120]. This importance of stratification within the current was explored in depth in the computational “turbidity current with a roof” model of Balachandar and colleagues [121]. These authors consider a downslope flow driven by suspended particles, similar to a turbidity current traveling downslope along the seafloor. However, unlike turbidity currents in the ocean which interact with the ambient water column, their model current (shown in Fig. 17) is bounded above by a “roof” that effectively prevents the entrainment of ambient water. The computational model encompasses the channel section bounded by the dashed lines. The authors employ periodic boundary conditions in the flow direction, based on the assumption that the mean flow in this direction does not vary. They furthermore assume that any sediment deposited at the bottom wall is immediately re-entrained into the flow, so that the overall amount of suspended sediment does not vary with time.

In spite of neglecting the energy loss and the dissipation experienced by oceanic currents through the entrainment of ambient water, the model provides interesting insight into the damping of turbulence due to self-stratification. The authors investigate the influence of the settling velocity \( u_s \) on the turbulence at \( Re = 180 \), where the Reynolds number is based on the friction velocity and the domain half height \( Re = uHL/hr \). Figure 18 shows the rms-values of the streamwise (\( u_{rms} \)) and wall-normal (\( w_{rms} \)) velocity fluctuations, respectively, as functions of the settling velocity, where both rms values have been normalized by the average friction velocity. Both velocity fluctuations are seen to decrease near the bottom wall for increasing settling velocity. Figure 19 shows that for even larger settling velocities the turbulence near the bottom wall is strongly damped, which is a consequence of the increasing local sediment concentration gradient. In summary, we can distinguish two regimes: for moderate settling velocities the flow remains turbulent, although with reduced turbulence intensity, whereas for large settling velocities the flow near the bottom wall completely relaminarizes.

The above configuration by Cantero et al. [121] employed a no-slip wall as the roof of the turbidity current, which continuously generates turbulence. This turbulence then diffuses toward the bottom wall, where it is being damped as a result of the stronger local stratification. Shringarpure et al. [122] extend this line of investigation and consider a free-slip boundary at the top wall, so that no turbulence is being generated there. This results in a complete suppression of turbulence across the entire water column for sufficiently high settling velocities. Figures 20 and 21 show isosurfaces of the swirling strength \( \lambda_z \) for two settling velocities, in order to identify vortical flow structures. Here, the swirling strength is defined as the imaginary part of the complex eigenvalues of the local velocity gradient tensor [123,124]. For the lower settling velocity case (\( u_s = 0.026 \)) shown in Fig. 20, vortical structures such as hairpin and quasi-streamwise vortices can be seen in the domain, although there are regions which are devoid of these vortical structures due to stratification effects. For the slightly higher settling velocity case (\( u_s = 0.0265 \)) shown in Fig. 21, the domain contains far fewer vortical structures at this specific instant in time. In fact, at later times, even these vortical structures disappear as the flow completely relaminarizes.

Shringarpure et al. [122] provide an explanation for the suppression of turbulence when the settling velocity is above the critical value. They conduct a quadrant analysis [125] of the streamwise (\( u \)) and wall-normal velocity (\( w \)) fluctuations, by assigning these fluctuations to respective quadrants in the \( u, w \)-plane. For instance, a second quadrant (Q2) event corresponds to a negative streamwise and a positive wall-normal velocity fluctuation at a location. This can be interpreted as an “ejection” event, where the low speed fluid from the near-wall region is ejected into the bulk flow. The authors show that above the critical settling velocity, the stable stratification effectively suppresses such second quadrant (Q2) events, which results in fewer and weaker turbulent hairpin vortices, thereby causing them to lose their ability to autogenerate, and suppressing the mechanism that produces turbulence.

4.9 Schmidt Number Effects. With regard to oceanic applications, it is important to keep in mind that salinity in water has a Schmidt number of 700. The effective Schmidt number for sediment in water depends on the grain size [126,127] and can be even higher. As a result, salinity concentration gradients can be significantly steeper than velocity gradients, and sediment concentration gradients can be even steeper. The relative magnitude of these length scales can be estimated via the Batchelor scale \( \lambda_B \), which represents the smallest scale for a diffusing scalar. It is
defined as the ratio of the Kolmogorov length scale $\eta$ of the turbulent velocity field and the square root of the Schmidt number 

$$\eta = \left( \frac{\nu^3}{\varepsilon} \right)^{3/4}, \quad \lambda_B = \frac{\eta}{\sqrt{\text{Sc}}}$$  \hspace{1cm} (44)$$

where $\varepsilon$ denotes the local dissipation rate of the turbulent kinetic energy. Hence, the grid resolution required to resolve the smallest diffusive scales can be significantly smaller than the Kolmogorov scale, which drastically increases the computational effort. To keep the computational cost manageable, most DNS simulations to date assume $\text{Sc} = 1$. Necker et al. [77] find that the integral properties of turbidity currents are independent of the actual Sc value, as long as this value is not much smaller than one. For non-Boussinesq lock-exchange flows, Birman et al. [128] report that the influence of Sc-variations in the range of 0.2–5.0 is small. Bonometti and Balachandar [129] perform a parametric study in order to gain insight into the effects of the Sc value on the dynamics of gravity currents. Their results show that for low Re values the front velocity increases with Sc, whereas at high Re values the front velocity becomes largely independent of Sc. The authors furthermore find the size of the lobe-and-cleft structures to be independent of Sc. These results suggest that for many applications it
Fig. 18 Turbulence statistics as a function of the wall-normal coordinate for different settling velocities: case 0 \((u_s = 0)\), case 1 \((u_s = 5 \times 10^{-3})\), case 2 \((u_s = 10^{-3})\), case 3 \((u_s = 1.75 \times 10^{-2})\), case 4 \((u_s = 2 \times 10^{-2})\), case 5 \((u_s = 2.125 \times 10^{-2})\). (a) Streamwise component (c) vertical component (Reprinted with permission from Cantero et al. [121]. Copyright 2009 by the American Geophysical Union).

Fig. 19 Turbulence statistics as a function of the wall-normal coordinate for different settling velocities: case 6 \((u_s = 2.3 \times 10^{-2})\), case 7 \((u_s = 2.5 \times 10^{-2})\), case 8 \((u_s = 3 \times 10^{-2})\), case 9 \((u_s = 3.5 \times 10^{-2})\), case 10 \((u_s = 5 \times 10^{-2})\). (a) Streamwise component (c) vertical component (Reprinted with permission from Cantero et al. [121]. Copyright 2009 by the American Geophysical Union).
Fig. 20 Swirling strength isosurface \( \lambda_{ci} = 22 \) for \( u_s = 0.026 \). At this value of the settling velocity, strong streamwise and hairpin vortices are visible near the bottom wall, indicating the presence of vigorous turbulence (Reprinted with permission from Shringarpure et al. [122]. Copyright 2012 by Cambridge University).

can be justified to employ Sc values of unity. Note, however, that when double-diffusive effects [130] become important, it will be essential to account for the different diffusivities of the various scalars, such as in the investigation of double-diffusive sedimentation by Burns and Meiburg [131].

4.10 Non-Boussinesq Currents. When the density difference between the light and dense fluids exceeds a few per cent, non-Boussinesq effects become influential and need to be accounted for. Examples concern tunnel fires, powder snow avalanches and pyroclastic flows triggered by volcanic eruptions [132]. One- and two-layer shallow water models for such flows were developed by Ungarish [133,134]. They indicate that the symmetry between the light and dense fronts observed under Boussinesq conditions is destroyed by non-Boussinesq effects, so that strong differences between the two fronts emerge.

These findings are confirmed by the depth-resolving two-dimensional Navier–Stokes simulations of Birman et al. [128], which covered full-depth lock-exchange flows over slip walls for density ratios from 0.2 to 0.998. The simulations show that for strong density contrasts the light front continues to resemble an energy-conserving current with a thickness close to half the channel height. The dense front, on the other hand, has a much smaller thickness and is strongly dissipative. Lowe et al. [135] performed corresponding laboratory experiments for non-Boussinesq currents, and compared their front velocity results with numerical simulation data of Birman et al. [128], as well as various theoretical predictions. Figure 22 illustrates the front speed of the dense current as a function of the density ratio \( \gamma \). The figure shows good agreement between the numerical results of Birman et al. and the experiments of Lowe et al. [135]. The simulation results of Bonometti et al. [139] show that for strong density contrasts the dissipation near the bottom wall can be an order of magnitude larger than at the interface and near the top wall. Bonometti et al. [79] furthermore show that the current speed during the slumping phase for heavy non-Boussinesq currents becomes independent of the lock aspect ratio \( \lambda \) even at very low values of \( \lambda \). However, for light non-Boussinesq currents the current speed during the slumping phase increases with \( \lambda \) for \( \lambda \leq 20 \).

4.11 Effects of Particle Inertia. Recall that the assumption of negligible particle inertia enabled us to obtain the velocity of the sediment concentration field by superimposing the settling velocity onto the fluid velocity field. This approach, which has been commonly taken in turbidity current simulations to date, has the immediate consequence that the sediment concentration field is divergence-free, so that particles do not get ejected from vortex cores, or accumulate near stagnation points. There are situations, however, when particle inertia will be of some importance, such as in powder snow avalanches or pyroclastic flows. Inertial effects can result in a host of new phenomena, as discussed for some canonical flows such as mixing layers or homogeneous turbulence in Refs. [140–144] and other investigations. Balachandar and colleagues developed an equilibrium Eulerian approach based on an expansion for small but finite particle Stokes numbers [145] that allows for the incorporation of the leading order effects of particle inertia, while retaining the Eulerian description of the sediment concentration field. Cantero et al. [146] apply this approach to explore the influence of particle inertia in gravity and turbidity

Fig. 21 Swirling strength isosurface \( \lambda_{ci} = 22 \) for \( u_s = 0.0265 \). At this slightly larger value of the settling velocity, the turbulence is strongly damped, so that the presence of hairpin vortices is greatly reduced (Reprinted with permission from Shringarpure et al. [122]. Copyright 2012 by Cambridge University).

Fig. 22 Front velocity of a dense, non-Boussinesq current as function of the density ratio \( \gamma \). Theoretical predictions are plotted as lines, while experimental and simulation data are represented by symbols. The theoretical prediction of Rotunno et al. [136] is shown as a solid line, and the theoretical prediction of Lowe et al. [135] is displayed as a dash-dotted line. Diamonds: experimental results of Lowe et al. [135]; squares: experimental data of Grobelbauer et al. [137]; asterisks: results of Keller and Chyou [138]; triangles: simulation data of Birman et al. [128]; circles: simulation data of Bonometti et al. [139]; plus symbols: simulation data of Rotunno et al. [136]. Figure based on data from Lowe et al. [135] and Rotunno et al. [136].
currents. Their two-dimensional simulations confirm that particles get ejected out of the Kelvin–Helmholtz mixing layer vortices, while accumulating near the front and closer to the body of the current, which in turn affects the resulting deposit profiles. This can result in elevated particle concentrations near the front and the bottom boundaries, and thus in larger front velocities. Furthermore, the shear stress at the bottom boundary is seen to increase, with potential consequences for the current’s ability to erode the sediment bed.

4.12 Effects of Rough Boundaries. Gravity and turbidity current simulations to date typically have assumed smooth boundaries. Geophysical applications, on the other hand, usually involve rough boundaries. The influence of such boundary roughness has recently been explored in the simulations by Bhaganganar [147], who employs an immersed boundary method to resolve the flow around roughness elements at Re = 4000. Here, the roughness elements in the near-wall region are represented as a periodic series of crests and valleys along the streamwise and spanwise directions. Comparisons of smooth and rough-wall gravity current results at identical Reynolds numbers indicate that for rough walls gravity currents travel more slowly and are subject to stronger lobe-and-cleft instabilities. Furthermore, the Kelvin–Helmholtz billows at the interface become three-dimensional faster, and strong three-dimensional turbulence is observed at earlier times. Özgökmen and Fischer [148] employ a spectral-element model to study gravity currents propagating downslope over both smooth and rough boundaries, in the presence of background stratification. Their results show that currents propagating over rough boundaries experience increased drag, which results in earlier separation from the bottom boundary as compared to currents propagating over smooth boundaries.

4.13 Effects of Viscosity Variations. Most turbidity current simulations assume Newtonian rheology and constant viscosity, even in the presence of sediment. It is well known, however, that at larger concentrations the sediment can alter the rheology of the fluid by introducing non-Newtonian behavior and/or a concentration dependent viscosity ([149]), with significant consequences for the dynamics of the flow. A striking example of the potential effects of viscosity variations on the flow is provided by Govindarajan and Sahu [150], who discuss their influence on the stability of shear flows. Along similar lines, Sameen and Govindarajan [151] study the effects of wall heating and the associated changes in viscosity, heat diffusivity and buoyancy on the stability of channel flows. Their results show that the flow is stabilized when the viscosity decreases toward the wall, while an opposite effect is observed when the viscosity increases toward the wall. Zonta et al. [152] investigate the effects of temperature-dependent viscosity in Newtonian fluids for forced convection in a channel where the fluid density is uniform. The authors show that the turbulence is enhanced on the cold side of the channel where the viscosity is higher. Zonta et al. [153] explore the effects of temperature-dependent viscosity on stably stratified channel flow when the fluid density varies with temperature. The authors find that the flow relaminarizes on the cold side of the channel where the viscosity is higher.

Yu et al. [154] employ a channel flow configuration to investigate the effects of viscosity variations on the dynamics of sediment laden flows. Toward this end, they study the following four cases. In case 0, the flow is driven by an applied streamwise pressure gradient without any sediment. In case 1, the sediment induces a density stratification which interacts with the flow dynamics. In cases 2 and 3, in addition to the density stratification, the sediment increases the viscosity, thereby damping the turbulence. For cases 2 and 3, the authors assume a Newtonian rheology with the viscosity varying as a function of the sediment concentration. Specifically, they employ the power-law rheological model proposed by Krieger and Dougherty [155]. They choose the power-law parameters such that the increase in viscosity for case 3 is higher than the increase in viscosity for case 2. Figure 4(c) in Yu et al. [154] shows the effective viscosity profile for the various cases studied. For case 3, the effective viscosity increases by 40% near the bed and by 15% near the top wall. This increased viscosity damps the turbulence fluctuations, as can be seen from the RMS of the velocity fluctuation profiles shown in Fig. 5 by Yu et al. [154]. The velocity fluctuations along the spanwise and vertical directions are more strongly damped as compared to the fluctuations along the streamwise direction. As the turbulence becomes weaker due to the enhanced viscosity, the sediment accumulates near the bed and its profile starts to resemble the laminar profile shown in Fig. 4(b) by Yu et al. [154]. Yu et al. [156] study the sediment transport in an oscillatory bottom boundary layer. When they consider an increase in viscosity due to sediment concentration, the turbulence in the flow is further damped as compared to cases where the turbulence damping occurs primarily due to density stratification. They also show that the enhanced viscosity, in addition to the density stratification, can cause laminarization of the bottom boundary layer.

5 Open Questions and Outlook

While the modeling of compositional gravity currents and dilute, noneroding turbidity currents has reached a certain level of maturity, the same cannot yet be said about dense turbidity currents with significant erosion, resuspension, and bedload transport. Especially the dynamics of the near-bed region of such currents, which is characterized by high sediment concentrations, is still poorly understood, as it is governed by intense particle–fluid and particle–particle interactions that give rise to strongly non-Newtonian dynamics, and to mass and momentum exchanges between the current and the sediment bed. As a result, insight into the erosional and depositional behaviors of such currents, and the coupling between the motion of the current above the sediment bed and the fluid flow inside the bed is just beginning to emerge. From the perspective of computational research into these matters, grain-resolving simulations appear to hold great potential in this regard. Approaches along these lines have been developed by several research groups in recent years, among them [157–159], and their application to turbidity current research can be expected to result in substantial progress.

A further interesting research direction can be found in the so-called inverse modeling of turbidity currents, which refers to the a posteriori reconstruction of the turbidity current flow from sparse information about the turbidite deposit it formed. The practical application in the context of deep-water oil exploration involves using incomplete deposit information obtained from isolated exploratory wells, in order to reconstruct the flow field during the early stages of the turbidity current. This early flow field can then serve as initial condition for a forward-in-time simulation that provides comprehensive information on the resulting deposit. First steps in this direction were taken by the investigation of Lesshafft et al. [160], based on a derivative-free surrogate modeling approach, albeit for much simplified conditions.

Several interesting aspects related to the modeling of gravity and turbidity currents have not been addressed in this review. Among these are so-called intrusions, i.e., gravity currents propagating horizontally along intermediate density contours in stratified ambient fluids [161,162]. These can be encountered both in the ocean and in the atmosphere, and similarly to gravity currents propagating along the base of a stratified ambient, they can trigger and interact with internal gravity waves, thereby giving rise to complex flow patterns and mixing dynamics [163–165]. The effects of rotation become important for large-scale gravity and turbidity currents in the ocean [166]. Due to Coriolis forces resulting from Earth’s rotation, such currents can develop a significant along slope component, rather than propagating mainly in the downslope direction, which fundamentally alters their dynamics, as well as the deposit patterns they generate. A more detailed
discussion of intrusions, along with further references, is provided by Ungarish [2].

Gravity currents in porous media [167,168] have received renewed interest in recent years, due to their importance in the context of CO2-sequestration in depleted oil reservoirs [169]. These currents frequently are amenable to computational modeling based on Darcy’s law and its variations, rather than the Navier–Stokes equations.

Low Reynolds number, viscous gravity currents [170] are encountered in both geophysical and industrial contexts. Examples of the former include gravity-driven flows in magma chambers as well as lava flows [171]. These are subject to solidification, which gives rise to additional complications in terms of developing accurate theoretical and computational models. Industrial applications of viscous gravity currents can be found in the petroleum industry, and specifically in processes related to oil well operations.

It is abundantly clear from the above discussion that the exploitation of gravity and turbidity currents represents a rich and vibrant research area that is characterized by fascinating fluid dynamical phenomena covering a wide range of length and time scales. The field gives rise to numerous questions of fundamental interest that remain unanswered to date, and whose resolution will require the joint efforts of experimental, theoretical, and computational scientists, along with field observations.

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