

## Model Predictive Control: Theory, Computation, and Design

### 2nd Edition

#### Errata for Second Edition, First Printing

Check [www.chemengr.ucsb.edu/~jbraw/mpc](http://www.chemengr.ucsb.edu/~jbraw/mpc) for a current list

February 3, 2019

1. Page 52, Figure 1.6. Change  $L_x$  and  $L_d$  to  $\tilde{L}_x$  and  $\tilde{L}_d$ , respectively, in the figure, and add the following sentence to the caption, "For simplicity we show the steady-state Kalman predictor form of the state estimator where  $\hat{x} := \hat{x}(k | k-1)$  and  $\tilde{L}_x := AL_x + B_dL_d$  and  $\tilde{L}_d := L_d$ ." Thanks to Pratyush Kumar and Travis Arnold of UW for pointing out this erratum.
2. Page 93, Equation (2.1). Add  $f : \mathbb{X} \times \mathbb{U} \rightarrow \mathbb{X}$  and that sets  $\mathbb{X}$  and  $\mathbb{U}$  are assumed closed.
3. Page 97, 9 lines from top, change (2.7) to (2.5).
4. Page 97, 10 lines from bottom. Change

In by far the majority of applications the control is constrained. Nevertheless, it is of theoretical interest to consider the case when the optimal control problem has no constraints on the control.

to

In by far the majority of applications the set of controls  $\mathbb{U}$  is bounded. Nevertheless, it is of theoretical interest to consider the case when  $\mathbb{U}$  is not bounded; e.g., when the optimal control problem has no constraints on the control.

5. Page 97, 4 lines from bottom. Change from:

$$\tilde{\mathcal{U}}_N^c(x) := \{\mathbf{u} \mid V_N(x, \mathbf{u}) \leq c\}$$

to:

$$\tilde{\mathcal{U}}_N^c(x) := \{\mathbf{u} \in \mathcal{U}_N(x) \mid V_N(x, \mathbf{u}) \leq c\}$$

6. Page 98, Assumption 2.3, lines 1-2. Change "If there are control constraints,  $\mathcal{U}_N(x)$  is defined ..." to "The set  $\mathcal{U}_N(x)$  is defined ..."
7. Page 98, Assumption 2.3, line 4. Change "If there are no control constraints ( $\mathbb{Z} = \mathbb{X} \times \mathbb{R}^m$ )," to "If  $\mathbb{U}$  is unbounded."
8. Page 98, first line of part (b) of proof. Change "If there are control constraints," to "If  $\mathbb{U}$  is bounded."
9. Page 98, fifth line in part (b) of proof. Change

If instead there are no control constraints, that  $\tilde{\mathcal{U}}_N^c(x)$  is closed follows from the fact that  $V_N(\cdot)$  is continuous.

to

If instead  $\mathbb{U}$  is unbounded, the set  $\tilde{\mathcal{U}}_N^c := \{\mathbf{u} \mid V_N(x, \mathbf{u}) \leq c\}$  for  $c \in \mathbb{R}_{>0}$  is closed for all  $c$  and  $x$  because  $V_N(\cdot)$  is continuous;  $\tilde{\mathcal{U}}_N^c(x)$  is the intersection of this set with  $\mathcal{U}_N(x)$ , just shown to be closed. So  $\tilde{\mathcal{U}}_N^c(x)$  is the intersection of closed sets and is closed.

10. Page 98, lines 7-12 from bottom. Change  $\mathcal{U}_N^c(x)$  to  $\tilde{\mathcal{U}}_N^c(x)$  (five places).
11. Page 98, 5 lines from bottom. Change  $\tilde{\mathcal{U}}_N(x)$  to  $\tilde{\mathcal{U}}_N^c(x)$ .
12. Page 104, Theorem 2.7. Replace "Assumptions 2.2 and 2.3" with "Assumptions 2.2 and 2.3 ( $\mathbb{U}$  bounded)."
13. Page 108, first sentence, second paragraph. Change "For all  $j \in \mathbb{0}_{0:N-1}$ , let  $V_j(x, \mathbf{u})$ ,  $\mathcal{U}_j(x)$ ,  $\mathbb{P}_j(x)$ , and  $V_j^0(x)$  be defined, respectively, by (2.3), (2.4), (2.5), and (2.6), with  $N$  replaced by  $j$ ." to:  
"For all  $j \in \mathbb{0}_{0:N-1}$ , let  $V_j(x, \mathbf{u})$ ,  $\mathcal{U}_j(x)$ ,  $\mathbb{Z}_j$ ,  $\mathbb{P}_j(x)$  (and  $V_j^0(x)$ ) be defined, respectively, by (2.3), (2.5), (2.6), and (2.7), with  $N$  replaced by  $j$ ."
14. Page 110, Proposition 2.10. Replace "Assumptions 2.2 and 2.3" with "Assumptions 2.2 and 2.3 ( $\mathbb{U}$  bounded)."
15. Page 110, Proposition 2.10 (a), first line. Change " $V_j(\cdot)$  is continuous in  $\mathbb{Z}_j$ ," to " $V_j(\cdot)$  is continuous in  $\mathbb{Z}_j$ ."
16. Page 110, Proposition 2.10 (b), last sentence. Change "In addition, the set  $X_N$  is positive invariant for  $x^+ = f(x, \kappa_N(x))$ ." to "In addition, the sets  $X_j$  and  $X_{j-1}$  are positive invariant for  $x^+ = f(x, \kappa_j(x))$  for  $j \in \mathbb{1}_{\geq 1}$ ."
17. Page 110, proof of Proposition 2.10 (b), last sentence. Change "That  $X_N$  is positive invariant for  $x^+ = f(x, \kappa_N(x))$  follows from (2.10), which shows that  $\kappa_N(\cdot)$  steers every  $x \in X_N$  into  $X_{N-1} \subseteq X_N$ " to "That  $X_j$  is positive invariant for  $x^+ = f(x, \kappa_j(x))$  follows from (2.10), which shows that  $\kappa_j(\cdot)$  steers every  $x \in X_j$  into  $X_{j-1} \subseteq X_j$ . Since  $X_{j-1} \subseteq X_j$ ,  $\kappa_j(\cdot)$  also steers every  $x \in X_{j-1}$  into  $X_{j-1}$ , so  $X_{j-1}$  is positive invariant under control law  $\kappa_j(\cdot)$  as well."
18. Page 111, proof of Proposition 2.10 (c), lines 5-10. Replace the following

this is possible since  $x_i \in X_j$  implies  $x_i \in \mathbb{X} := \{x \in \mathbb{R}^n \mid \mathbb{U}(x) \neq \emptyset\}$ . Since  $Z_j$  is closed, there exists a subsequence, indexed by  $\mathbb{1}$ , such that  $z_i = (x_i, u_i) \rightarrow \bar{z} = (\bar{x}, \bar{u}) \in Z_j$  as  $i \rightarrow \infty$ ,  $i \in \mathbb{1}$ . But  $X_j = \{x \in \mathbb{X} \mid \exists u \in \mathbb{U}(x) \text{ such that } f(x, u) \in X_{j-1}\} = \{x \in \mathbb{X} \mid \exists u \in \mathbb{U}(x) \text{ such that } (x, u) \in Z_j\}$  (see (2.11)). Hence  $\bar{x} \in X_j$  so that  $X_j$  is closed.

with the following

this is possible since  $x_i \in X_j$  implies  $x_i \in \{\mathbb{X} \mid \mathcal{U}_j(x) \neq \emptyset\}$ . Since  $\mathcal{U}_j(x) \subseteq \mathbb{U}$  and  $\mathbb{U}$  is bounded, by the Bolzano-Weierstrass theorem there exists a subsequence, indexed by  $\mathbb{1}$ , such that  $u_i \rightarrow \bar{u}$  (and  $x_i \rightarrow \bar{x}$ ) as  $i \rightarrow \infty$ ,  $i \in \mathbb{1}$ . The sequence  $(x_i, u_i) \in Z_j$ ,  $i \in \mathbb{1}$  converges, and, since  $Z_j$  is closed,  $(\bar{x}, \bar{u}) \in Z_j$ . Therefore  $f(\bar{x}, \bar{u}) \in X_{j-1}$  and  $\bar{x} \in X_j$  so that  $X_j$  is closed.

19. Page 111, first line of part (d). Change  $X_f$  to  $\mathbb{X}_f$ .
20. Page 111, 11 lines from the bottom. Change “ $f^{-1}(\cdot)$  is bounded on bounded sets if  $f(x, u) = Ax + Bu$  and  $A$  is nonsingular or if  $f(\cdot)$  is Lipschitz in  $x$ ,” to “ $f^{-1}(\cdot)$  is bounded on bounded sets if  $\mathbb{U}$  is bounded and either  $f(x, u) = Ax + Bu$  and  $A$  is nonsingular, or  $f_c(x, u)$  is Lipschitz in  $x$ .”
21. Page 114, Assumption 2.14 (a). Change  $\ell(x, u)r$  to  $\ell(x, u)$  in the last inequality.
22. Page 115, Proposition 2.15. Replace “Assumptions 2.2 and 2.3” with “Assumptions 2.2 and 2.3 ( $\mathbb{U}$  bounded).”
23. Page 115, 2 lines before Proposition 2.16. Change  $X_f$  to  $\mathbb{X}_f$ .
24. Page 115, Proposition 2.16. Replace “Assumptions 2.2 and 2.3” with “Assumptions 2.2 and 2.3 ( $\mathbb{U}$  bounded).”
25. Page 116, 3 lines from bottom. Change  $\mathbb{U}^N$  to  $\mathcal{U}_N(x)$ . Thanks to Doug Allan of UW for pointing out this erratum.
26. Page 117, 4 lines from top. Delete the sentence, “Since there are no state or terminal constraints, the state sequence  $\tilde{\mathbf{x}}$  is clearly feasible if  $u \in \mathbb{U}$ .”
27. Page 117, 6 lines from bottom. Delete the repeated phrase, “if, for all  $x \in \mathbb{X}_f$ , there exists a  $u$  such that” before the displayed equation.
28. Page 118, Proposition 2.18. Replace “Assumptions 2.2 and 2.3” with “Assumptions 2.2 and 2.3 ( $\mathbb{U}$  bounded).”
29. Page 118, second displayed equation. Change  $X_f$  to  $\mathbb{X}_f$ .
30. Page 119, 11 lines from top. Change the phrase

The monotonicity property can be used to establish the descent property of  $V_N^0(\cdot)$  proved in Theorem 2.19 by noting that

to:

The monotonicity property can also be used to establish the (previously established) descent property of  $V_N^0(\cdot)$  by noting that

31. Page 119, Theorem 2.19. Replace “ $X_N$ ” with “ $X_N(\bar{X}_N^c, \text{ for each } c \in \mathbb{R}_{>0})$ ” (two places).
32. Page 120. Insert at top of page.

For the proof with  $\mathbb{U}$  unbounded, note that the lower bound and descent condition remain satisfied as before. For the upper bound, if  $\mathbb{X}_f$  contains the origin in its interior, we have that, since  $V_f(\cdot)$  is continuous, for each  $c > 0$  there exists  $0 < \tau \leq c$ , such that  $\text{lev}_\tau V_f$  contains a neighborhood of the origin and is a subset of both  $\mathbb{X}_f$  and  $\bar{X}_N^c$ . One can then show that  $V_N^0(\cdot) \leq V_f(\cdot)$  for each  $N \geq 0$  on this sublevel set, and therefore  $V_N^0(\cdot)$  is continuous at the origin so that again Proposition B.25 applies, and Assumption 2.17 is satisfied on  $\bar{X}_N^c$  for each  $c \in \mathbb{R}_{>0}$ .

33. Page 122, Proposition 2.24. Replace “ $X_N$ ” with “ $X_N(\bar{X}_N^c, \text{ for each } c \in \mathbb{R}_{>0})$ ” (two places).
34. Page 141, first line of quoted material. Change  $X_f$  to  $\mathbb{X}_f$ .
35. Page 146, 9 lines from top. Replace  $i \in \mathbb{I}_{i:N-1}$  with  $j \in \mathbb{I}_{i:N-1}$ . Thanks to Danylo Malyuta of U. Washington for pointing out this erratum.
36. Page 274, Definition 4.8. Change the phrase

The estimate is RGAS if for all  $x_0$  and  $\bar{x}_0$ , and bounded  $(\mathbf{w}, \mathbf{v})$ , there exist functions  $\alpha(\cdot) \in \mathcal{KL}$  and  $\delta_w(\cdot) \in \mathcal{K}$  such that

to

The estimate is RGAS if there exist functions  $\alpha(\cdot) \in \mathcal{KL}$  and  $\delta_w(\cdot) \in \mathcal{K}$  such that for all  $x_0$  and  $\bar{x}_0$ , and bounded  $(\mathbf{w}, \mathbf{v})$

Thanks to Doug Allan of UW for pointing out this erratum.

37. Page 329, Exercise 4.7, 3rd line. Change, “let  $\gamma(\cdot)$  be any  $\mathcal{K}$  function,” to “let  $\gamma(\cdot)$  be any  $\mathcal{K}$  function and  $a_i \in \mathbb{R}_{\geq 0}, i \in \mathbb{I}_{1:n}$ .”
38. Page 366, Exercise 5.5. Change  $d_H(\mathbb{A} \oplus \mathbb{C}, \mathbb{B} \oplus \mathbb{C}) = d_H(\mathbb{A}, \mathbb{B})$  to  $d_H(\mathbb{A} \oplus \mathbb{C}, \mathbb{B} \oplus \mathbb{C}) \leq d_H(\mathbb{A}, \mathbb{B})$ . Thanks to Dr. Saša V. Raković for pointing out this erratum.
39. Page 366, Exercise 5.5. Delete the phrase, “satisfying  $\mathbb{B} \subseteq \mathbb{A}$ .”
40. Page 366, Exercise 5.6. Delete the phrase, “satisfying  $\mathbb{A} \subseteq \mathbb{B}$ .”
41. Page 366, Exercise 5.8. Replace  $+$  with  $\oplus$  (two places).
42. Page 527, 5 lines from bottom. Change “plus  $n$  forward sweeps,” to “plus  $m$  forward sweeps.”