function (2.12). We do this using the $\mathcal{H}_2$ norm, that is

$$\|\mathcal{H}_r\|_2^2(k_z) := \frac{1}{2\pi} \int_{-\infty}^{\infty} \|\mathcal{H}_r(k_z, \omega)\|_H^2 S \, d\omega,$$

for $r = u, v, w$, and $s = x, y, z$.

We then investigate the dependence of each of the subsystems on the Reynolds number. We analytically demonstrate that amplification from both spanwise and wall-normal forcing to streamwise velocity is $O(R^2)$, while amplification for all other components is $O(R)$.

Application of the temporal Fourier transform allows us to represent system (4.1) in terms of its block diagram shown in figure 6 with $\Omega := \omega R$. We note that the same temporal scaling was previously employed by Gustavsson (1991) in his undertaking to determine the transient growth of the wall-normal vorticity. Thus, from this block diagram it follows that for the streamwise constant perturbations the frequency responses from $d_y$ and $d_z$ to $u$ scale as $R^2$, whereas the responses from all other inputs to other velocity outputs scale at most linearly with $R$. In particular, at $k_z = 0$ the streamwise forcing does not influence the wall-normal and the spanwise velocity components. It is noteworthy that the coupling term $C_p = -ik_zU'$ is crucial for providing this $R^2$-scaling. Namely, one can observe from figure 6 that in the absence of shear or spanwise variations in the wall-normal velocity perturbation all components of operator $\mathcal{H}(0, k_z, \omega)$ in (2.12) are at most proportional to $R$. This further exemplifies the importance of the vortex stretching mechanism (Landahl 1975) in the wall-bounded shear flows. We remark that the numerical experiments of Kim & Lim (2000) indicated that without the coupling from $v$ to $\omega_y$ the near-wall turbulence decays in a fully turbulent channel flow.

We now state the main result whose proof can be found in Appendix D. Theorem 1 quantifies the energy amplification for each of the components of frequency response (2.12) at $k_z = 0$.

**Theorem 1.** For any streamwise constant channel flow with nominal velocity $U(y)$, the $\mathcal{H}_2$ norms of operators $\mathcal{H}_r(k_z, \omega, R)$ that map $d_s$ into $r$, $\{r = u, v, w; s = x, y, z\}$, are given by

$$\begin{bmatrix}
||\mathcal{H}_{ux}||_2^2(k_z) & ||\mathcal{H}_{uy}||_2^2(k_z) & ||\mathcal{H}_{uz}||_2^2(k_z) \\
||\mathcal{H}_{vx}||_2^2(k_z) & ||\mathcal{H}_{vy}||_2^2(k_z) & ||\mathcal{H}_{vz}||_2^2(k_z) \\
||\mathcal{H}_{wx}||_2^2(k_z) & ||\mathcal{H}_{wy}||_2^2(k_z) & ||\mathcal{H}_{wz}||_2^2(k_z)
\end{bmatrix}
= \begin{bmatrix}
f_{ux}(k_z)R & g_{uy}(k_z)R^3 & g_{uz}(k_z)R^3 \\
0 & f_{vy}(k_z)R & f_{vz}(k_z)R \\
0 & f_{wy}(k_z)R & f_{wz}(k_z)R
\end{bmatrix},$$

(4.2)

where the $f$ and $g$ functions are independent of $R$.

The energy amplification of streamwise constant perturbations scales as $R^3$ from the forces in the wall-normal and the spanwise directions to the streamwise velocity. In all other cases it scales at most as $R$. In particular, at $k_z = 0$ the streamwise forcing does not influence the wall-normal and the spanwise velocity components. This further illustrates the dominance of the streamwise velocity perturbations and the forces in the remaining two spatial directions for high Reynolds number channel flows.

The expressions for the terms that multiply $R$ in (4.2) are the same for all channel flows, because these terms depend only on nominal-velocity independent operators $\mathcal{L}$ and $\mathcal{S}$ (see Appendix E for details). On the