local error \( v^5 \) which in this case is given by

\[
V^{\text{loc}}_v = -\frac{1}{4\beta} \sum_{n \neq 0, \ n \in \mathbb{Z}^d_N} \frac{\hat{O}_n}{(g_o + \hat{O}_n)(f_o + \hat{O}_n)}.
\]  

In the case of relative position and velocity error feedback, which corresponds to \( g_o = 0 \) and \( f_o = 0 \), the sum in Eq. (36) becomes \(-\sum 1/\hat{O}_n\). This has the same form as \( V^v_c \) in Eq. (28) for the standard consensus problem, and thus will grow asymptotically as derived in Eq. (34). For this scenario, the final answer is listed as \( V^{\text{loc}}_v \) in Table I after multiplying by the extra \( 1/\beta \) factor. In the case of relative position and absolute velocity feedback, the sum in Eq. (36) becomes \( \sum -1/(f_o + \hat{O}_n) \). Each term is bounded from above by \(-1/(f_o + \hat{O}_n) \leq -1/f_o \) since \( f_o < 0 \) and \( \hat{O}_n \leq 0 \). Thus the entire sum has an upper bound that scales like \( M \), which yields a constant bound for the individual local error once divided by the network size \( M \). An exactly symmetric argument applies to the case of absolute position but relative velocity feedback. Finally, in the case of both absolute position and velocity feedback \( f_o < 0 \) and \( g_o < 0 \) implying a uniform bound on each term in the sum. Similarly the entire sum scales like \( M \) and thus is uniformly bounded upon division by the network size. All of these four cases for the local error scalings are summarized in Table I.

We now consider the case of the deviation from average measure (31) which for our specific algorithm is

\[
V^{\text{dav}}_v = \frac{d}{2} \sum_{n \neq 0, \ n \in \mathbb{Z}^d_N} \frac{1}{(g_o + \hat{O}_n)(f_o + \hat{O}_n)}.
\]

When \( g_o < 0 \) and \( f_o < 0 \), each term in the sum is bounded and the entire sum scales as \( M \). Thus, the individual deviation from average at each site is bounded in this case. When either \( f_o = 0 \) or \( g_o = 0 \), then the sums scale like \(-\sum 1/\hat{O}_n\) (since the other factor in the fraction is uniformly bounded), i.e. like the deviation from average in the consensus case (34).

The only case that requires further examination is that of relative position and relative velocity feedback (\( g_o = f_o = 0 \)). In this case

\[
V^{\text{dav}}_v = \frac{d}{2} \sum_{n \neq 0, \ n \in \mathbb{Z}^d_N} \frac{1}{(g_o + \hat{O}_n)(f_o + \hat{O}_n)} \leq \frac{d^2}{2\beta^2} \frac{1}{N^d} \sum_{n \neq 0, \ n \in \mathbb{Z}^d_N} \frac{1}{(n_1^2 + \cdots + n_d^2)^{\frac{1}{2}}},
\]

where the inequality is derived by the same argument used in deriving the inequality (33). Dividing this expression by the network size \( N^d \) and using the asymptotic expressions (52) yields

\[
\frac{V^{\text{dav}}_v}{N^d} \leq C_d \frac{1}{\beta^2} \left\{ \frac{1}{\log(N)} \right\} \frac{1}{(1 - N^{4-d})^d} \quad d \neq 4,
\]

where \( C_d \) is a constant depending on the dimension \( d \) but independent of \( N \) or the algorithm parameter \( \beta \). Rewriting these bounds in terms of the total network size \( M = N^d \) gives the corresponding entries in Table I, where the other cases are also summarized.

We finally point out that \( V^{\text{dav}}_v \leq 4V^{\text{dav}}_v \) due to an argument identical to that employed in the consensus case. We thus conclude that the upper bounds just derived apply to the case of the long range deviation measure as well.

The role of viscous friction: It is interesting to observe that in vehicular models with viscous friction (6), a certain amount of absolute velocity feedback is inherently present in the dynamics. The model (6) with a feedback control of the form (8) has the following Fourier symbol for the velocity feedback operator \( F \)

\[
\hat{F}_n = -\mu + f_o + \hat{O}_n.
\]

We conclude that even in cases of only relative velocity error feedback (i.e. when \( f_o = 0 \)), the viscous friction term \( \mu > 0 \) provides some amount of absolute velocity error feedback. Thus, in an environment which has viscous damping, performance in vehicle formation problems scale in a similar manner to consensus problems. These comments are also applicable to the lower bounds developed in the next section.

<table>
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<tr>
<th>Consensus</th>
<th>Microscopic</th>
<th>Macroscopic</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1/\beta )</td>
<td>( 1/\beta )</td>
<td>( M )</td>
</tr>
<tr>
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<td>( 1 )</td>
<td>( d = 1 )</td>
</tr>
<tr>
<td>( \log(M) )</td>
<td>( d = 2 )</td>
<td></td>
</tr>
<tr>
<td>( d = 3 )</td>
<td>( d = 1 )</td>
<td></td>
</tr>
</tbody>
</table>

**Table I**: Summary of asymptotic scalings of upper bounds in terms of the the total network size \( M \) and the spatial dimensions \( d \). Performance measures are classified as either microscopic (local error), or macroscopic (deviation from average or long range deviation). There are four possible feedback strategies in vehicular formations depending on which combination of relative or absolute position or velocity error feedback is used. Quantities listed are up to a multiplicative factor that is independent of \( M \) or algorithm parameter \( \beta \).