MEASURING THE FREQUENCY RESOLUTION OF A MULTI DEGREE OF FREEDOM RESONATOR FOR MASS DETECTION

K.M. Moran and K.L. Turner
Dept. of Mechanical Engineering, UCSB, Santa Barbara, CA, USA

ABSTRACT

This work focuses on measuring resolution of a coupled, single input single output, multi degree of freedom (MDOF) resonator. This type of device can be operated as a chemical nose. Specifically, we show that the loss in resolution due to coupling does not degrade the signal beyond the usable spectrum. In fact, the sensing cantilevers in this prototype are able to detect as low as 1.47 fg mass sensitivity for a near ambient, table top experiment.

KEYWORDS

Single Input Single Output (SISO), MEMS, Resonator, Mass sensor, Frequency Resolution, chemical nose

INTRODUCTION

Microcantilevers are very useful transducers. Some applications include: atomic force microscopy [1], physical sensors [2, 3], mechanical force sensors [4], and parameter sensors such as mass sensors [5]. Due to miniaturization, the sensitivity of these transducers is greatly improved. This work focuses on the application of coupled arrays of microcantilevers as mass sensors for chemical detection.

A SISO design is studied [6], to be implemented as a chemical nose. The device studied is shown in Figure 1. It consists of a large, plate-like coupling beam, with several smaller microbeams protruding from the free end. Each microbeam can be chemically functionalized to sense a unique analyte. The entire system response is measured through the coupling beam via laser Doppler Vibrometry (LDV) or a single electrostatic output, thus requiring a single measurement, rather than N measurements for N number of sensing microbeams, which greatly simplifies the necessary equipment for measurement. Figure 2 shows the frequency response of the device acquired using LDV and piezo actuation. Simple CMOS fabrication techniques can be used to include on chip sensing, such as piezo electric, capacitive or electromagnetic.

THE SISO SENSOR

Transducer Design

The current device under test is a MDOF cantilever beam. A large, coupling cantilever extends from the substrate of the chip. Four sensing beams protrude from the free end of the coupling beam. Each sensing microbeam is tuned by length, in order to have a distinct natural frequency. Each mode is designed to be well spaced from the surrounding modes. Table 1 gives the geometry of each of the beams.

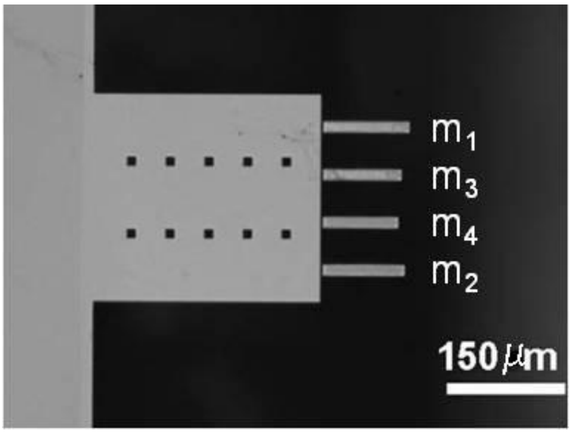

Figure 1: Scanning Electron Micrograph of the device under test. The large coupling mass is distinctly shown with vias through the center, and sensing microbeams are labeled m_1 through m_4.

Table 1: Number N of abstracts submitted and number M of submitting countries.

Beam	Length (µm)	Width (µm)	Thickness (µm)	Natural Frequency (kHz)
m_{coup}	335	217	10	53.4
m1	129	20	5	85.9
m2	124	20	5	97.1
m3	119	20	5	102.7
m4	114	20	5	113.6

Transducer Physics

The physics of the device are governed by the equations of motion presented in Equation (1).

$$\begin{aligned} &m_s\ddot{x} + c_s\dot{x} + k_s x + \sum_{i=1}^{N} c_i\left(\dot{x} - \dot{y}_i\right) + \sum_{i=1}^{N} k_i\left(x - y_i\right) = u(t) \\ &m_i\ddot{y}_i + c_i(\dot{y}_i - \dot{x}) + k_i(y_i - x) = 0 \end{aligned} \qquad (1)$$

This may be expanded to N degrees of freedom, but for the device presented here N=4. Each sensing beam is designed to have a well spaced fundamental frequency. The device response is shown in Figure 2. The device response is measured through the coupling mass (m_{coup}).

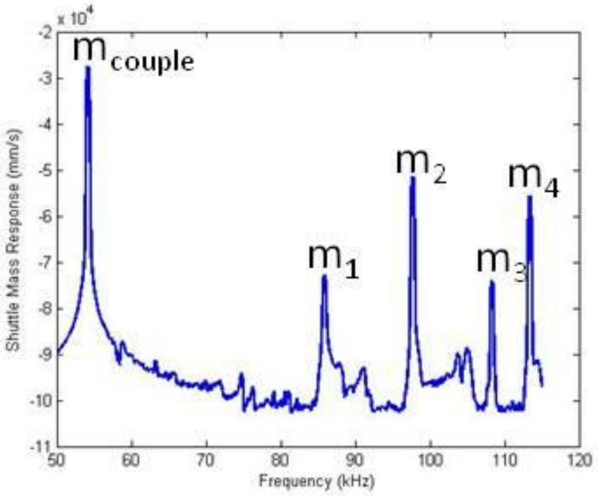

Figure 2: Frequency response of the N=4 MDOF device under test.

SISO MASS SENSING

The minimum mass detectable is the mass sensitivity of the transducer. Mass sensitivity can be approximated rather well using the relationship shown in Equation (2):

$$dM = R^{-1} d\omega \tag{2}$$

Here R is the mass responsivity, how much the sensor reacts to added mass; and $d\omega$ is the frequency resolution or how closely you can resolve the frequency of the sensor. For the coupled system, the responsivity is a matrix, rather than a scalar quantity. This tells us precisely how each resonator reacts to any change in mass throughout the system. However, the off diagonal entries are approximately an order of magnitude lower than the diagonal components, and therefore negligible for this study. Table 2 shows the diagonal components of the responsivity matrix.

Table 2: Diagonal entries of matrix of responsivity for the device under test, in Hz/pg

$\mathbf{R_{11}}$	-2170
$\mathbf{R_{22}}$	-2540
$\mathbf{R_{33}}$	-2810
$\mathbf{R_{44}}$	-3230

Figure 3 shows a characteristic plot of a mass loaded sensing cantilever, which has the largest frequency shift, and the accompanied shift of the neighboring resonators. The responsivity of the beams can be tuned during the design process. Conversely, the frequency resolution is not as easily predictable. The next section will discuss the frequency resolution of the SISO sensor.

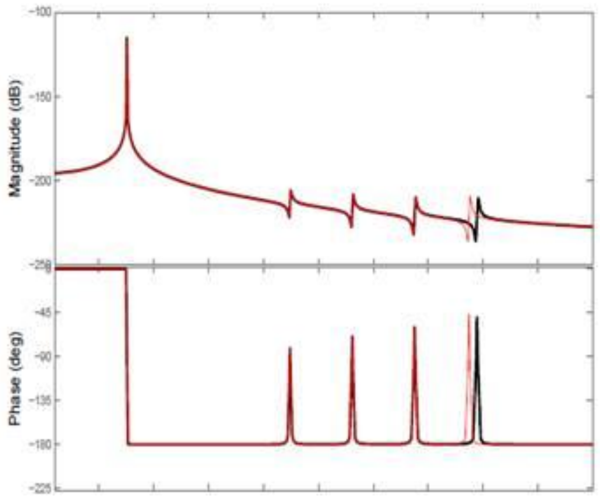

Figure 3: Example of the change in frequency response due to mass loading of sensing beam m_4. The black trace shows the device before loading, and red is the response after mass is added.

FREQUENCY RESOLUTION

The frequency resolution of the device depends on the noise present in the system. Previous work has focused on deriving the frequency resolution of sensors like this [7]. Here we present the frequency resolution as we find it experimentally. Prototype testing is done with the use of a Polytec Laser Doppler Vibrometry. On chip measurements can be implemented using electrostatic, electromagnetic, or piezoelectric methods.

Experimental Method

The device under test is contained within a vacuum chamber, held at 250Torr. The chamber rests on an air table to remove the low frequency noise from the environment. The laser is shined onto the device and the vibrometer output is recorded on a controlling computer. The experimental setup is shown in Figure 4.

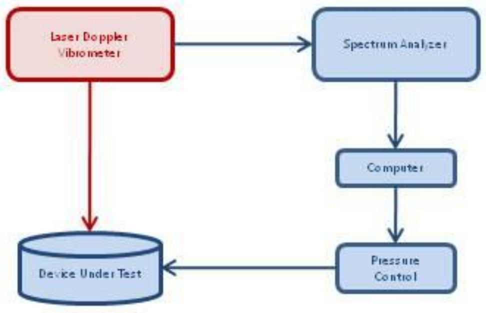

Figure 4: Cartoon of Experimental Setup

Frequency resolution data is collected for each of the resonators, as well as each sensing resonator's resolution as measured through the coupling mass. Figures 5 and 6 show the resolution data. For each scenario, the peak frequency is recorded every three seconds for three hours.

For the individual sensing beams, the laser rests on the beam for an hour prior to beginning measurements, in order to allow for spot heating to reach an equilibrium temperature, thus negating any drift effects. Each beam has an approximate Quality factor of 300.

Experimental Results

The frequency resolution of the transducer is calculated as the variance of the natural frequency from the data collected. Table 3 shows the single- and multi-degree of freedom resolution for the sensing beams.

Table 3: Resonator frequency resolution with direct measurement and with SISO system.

Resonator	Frequency Resolution (Hz)	MDOF Frequency Resolution (Hz)
m_1	63	91
m_2	2.93	3.15
m_3	0.59	1.6
m_4	1.9	2.23

The frequency resolution is combined with the calculated mass responsivity, to give the mass sensitivity for the device in a table top experiment. Mass Sensitivities are presented in Table 4. Due to the small size of the sensing resonators, we are still able to achieve acceptable mass sensitivity.

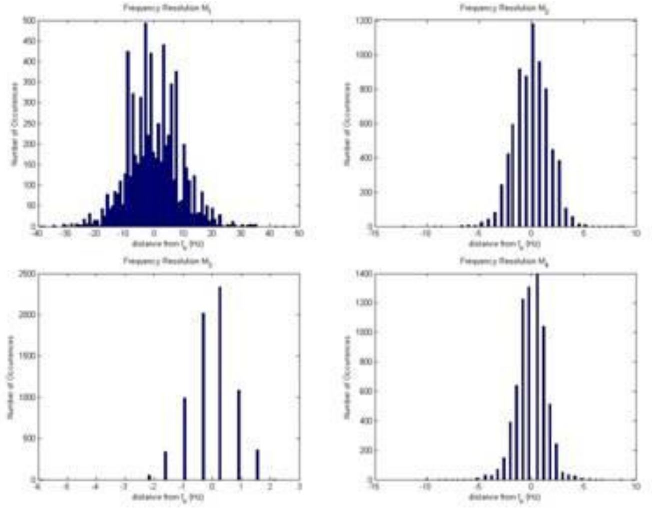

Figure 5: Frequency resolution histograms of m_1 (top, left), m_2 (top,right) m_3 (bottom, left),and m_4 (bottom, right), measured directly.

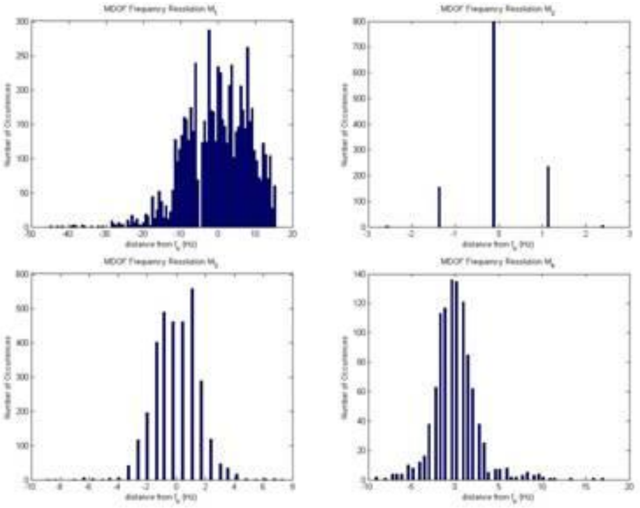

Figure 6: Frequency resolution histograms of m_1 (top, left), m_2 (top,right) m_3 (bottom, left),and m_4 (bottom, right), measured by the coupling resonator.

Table 4: The mass sensitivity for the sensing microbeams

Resonator	Mass Sensitivity (pg)	MDOF Mass Sensitivity (pg)
m_1	0.029	0.0419
m_2	1.2e-3	1.2e-3
m_3	2.10e-4	5.69e-4
m_4	5.88e-4	6.90e-4

CONCLUSION

Presented here is a SISO transducer, designed for use as a mass sensor. The frequency resolution is measured by finding the natural frequency and taking the variance of the experimentally found natural frequency for a given duration of time. The experimentally found frequency resolution is combined with the predicted mass responsivity to find the ultimate mass sensitivity of the transducer, in the present system. The mass sensitivities are on the order of femtogram resolutions, which is competitive for multidegree of freedom, MEMS size scale devices.

REFERENCES

[1] D. Rugar, & P. Hansma, “Atomic Force Microscopy,” Physics Today, Vol 43 (10), 1990.

[2] J. Barnes, R. Stephenson, M. Welland, C. Gerber, & J. Gimzewski, “Photothermal spectroscopy with femtojoule sensitivity using a micromechanical device”, Nature, 372, 79-81, 1994

[3] P. Oden, G. Chen, R. Steele, R. Warmack, & T. Thundat, “Viscous drag measurements utilizing microfabricated cantilevers”, Appl. Phys. Lett., 68, 3814-16, 1996.

[4] J. Harley, T. Kenny, “Piezoresistive cantilevers with femtonewtonforce resolution”, *Proceedings 10th conference on Solid-state sensors and actuators, Tranducers*, pp 1628-1631, 1999.

[5] T. Thundat, E. A. Wachter, S. L. Sharp, and R. J. Warmack, *Detection of mercury vapor using resonating microcantilevers*, Appl. Phys. Lett. 66, 1695, 1995.

[6] B. DeMartini, J. Rhoads, M. Zielke, K. Owen, S. Shaw, and K. Turner, “A single input-single output coupled microresonator array for the detection and identification of multiple analytes”, Applied Physics Letters, Vol. 93, 2008.

[7] K. Moran, K. Astrom, B. DeMartini, and K. Turner, “Frequency Resolution of a Multi Degree of Freedom Resonator,” *Proceedings 8th annual conference on Sensors,* 2009

CONTACT

* Kari Moran, tel: 1-805-893-5341;
klukes@engineering.ucsb.edu