Study of electrostatic interactions in Micro Electro Mechanical resonant oscillators

Rajashree Baskaran, Kimberly L. Turner
Department of Mechanical and Environmental Engineering, University of California, Santa Barbara

ABSTRACT

Resonant mode operation is common in many MicroElectroMechanical (MEM) applications including accelerometers, gyroscopes and filters [Kovacs (1998), Nguyen (1999)]. When electrostatic transduction is used in these applications, concerns about cross talk and fringing field effects due to geometry are major issues. In this paper, an electrostatically coupled system is briefly introduced, modeled and the dynamic response due to small parametric (displacement dependant) electrostatic force is analyzed using perturbation methods. The presence of coupled parametric resonance has a very significant effect on the dynamic response. Experimental verification of the occurrence of this phenomenon is also presented here. The coupled oscillator system can also be used as an in situ test device to understand the electrostatic parameters in a system. The method of modeling and analysis presented here is simple, yet captures the dynamic behavior of a system due to a small force. This method can be generalized and will be a useful tool in any resonant MEM system design.

Keywords: MEMS oscillator resonance coupled parametric multiple electrostatic

1. INTRODUCTION

Oscillators operating in a resonant mode are widely used for sensing and actuation in micro-systems. Low damping and unique scaling laws lead to very interesting effect in MEMS resonant dynamic systems [Turner (1998)]. There are numerous applications in filter technology, force sensing, switches and other sensing/actuation systems for resonant MEM systems. Electrostatics is a preferred mode of transduction in micro-systems. Modeling the electrostatic forces generated in complicated geometries is a challenge in such systems. Analytical methods can be time-consuming and problem specific. Using simulations is a good option, but is limited by lack of knowledge of material properties at the micro-scale and accurate geometries due to processing variations.

In this paper, we present a novel oscillator system whose measurable dynamic properties can be used to understand the electrostatic properties of the micro-scale transducers. This system also brings out the effect of electrostatic interaction (between two components in the same system or 'cross-talk' between two adjacent systems) in introducing a time-varying stiffness into the dynamics. Even if the electrostatic coupling strength is small, its effect on the system stability may be pronounced and need to be taken into account in design.

Section 2 begins with an introduction to the coupled oscillator system. In the first two sub sections the electrostatic and mechanical domain modeling is presented. The complete system dynamics are modeled and one of the cases of interest is analyzed using perturbation techniques. Section 3 describes some experimental observations from one such system and is followed by a discussion of the implications of the results for various applications in section 4. Section 5 summarizes and looks ahead at the future directions of work with coupled oscillator systems.

2. SYSTEM MODELING AND ANALYSIS

The system consists of two mechanically isolated torsional MEM oscillators. The oscillators are actuated using out-of-plane electrostatic drive, one of the common sensing/actuation mechanisms in micro-scale systems. The two oscillators are coupled by electrostatic interactions between adjacent comb fingers. Figure 1 is a schematic of the basic coupled oscillators system and Figure 2 shows the scanning electron micrograph of the coupling region. Typical dimension values are given in Table 1.The details of the design and fabrication of such a system are presented elsewhere[Baskaran (2000)].

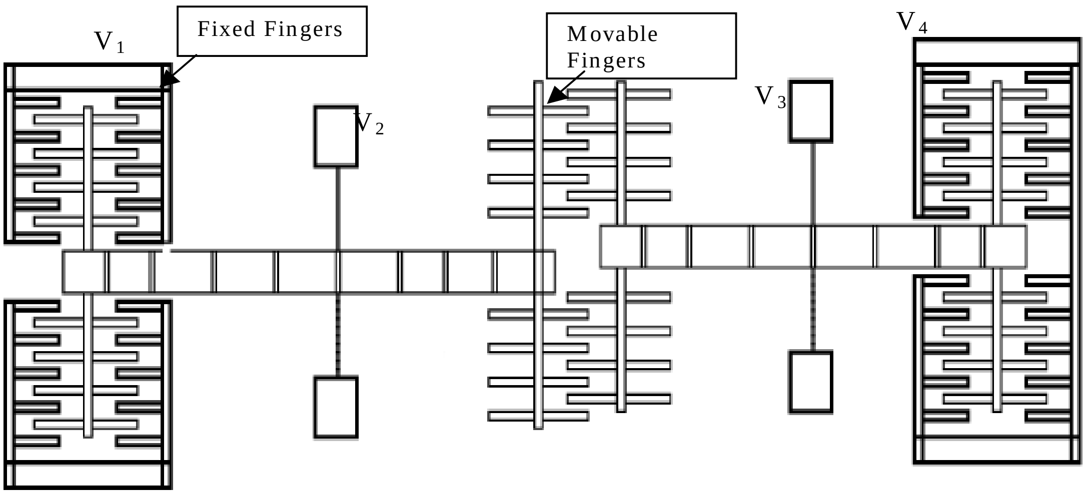

Figure 1: Schematic of the coupled oscillator system. The 'comb fingers' represented by dark lines are anchored to the substrate and the others are 'floating' or movable fingers.

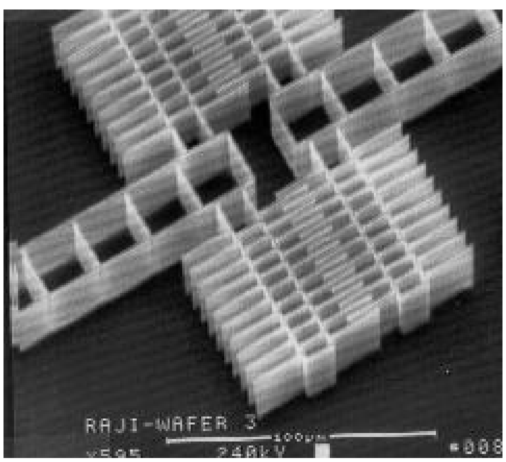

Figure 2: SEM of the coupling comb fingers after the process of deep etching and release. The depth to thickness aspect ratio of the 'released' or movable features is about 25:1.

Design Parameter	**Dimension in μm**
Thickness of all features	1
Depth of all features	25
Torsion beam length	38
Backbone length	360
Backbone width	24
Combfinger length	25
Combfinger gap	2

Table 1: Typical design values of the MEMS torsional oscillator.

In order to model the system, two energy domains need to be taken into account – the mechanical and electrostatic. The electrostatic interaction is modeled using a Boundary Element Method and mechanical parameters, the stiffness and inertia of the oscillators, are obtained by using a Finite Element model.

2.1 Electrostatic Model

Using analytical expressions to model the electrostatic forces generated in complex geometries like comb finger configuration is non-trivial and time-consuming [Hui (2000)] but has good agreement with Boundary Element simulations. The ratio of the torque generated to the angle of rotation at the comb fingers is defined as an 'electrostatic stiffness'. It has been observed that the distance to the ground plane in the finger configuration affects the out of plane force [Tang (1992), Miller (1997)] but has not been taken into account in many models. Boundary Element simulations of the out of plane forces give comparable results to analytical expressions and experimental observations [Hui (2000), Turner (1999)a]. Using a model shown in Figure 3, a boundary element electrostatic solver COULOMB (Integrated Engineering Software) was used to obtain the variation of the torque produced with changing comb finger vertical position (which translates to rotation angle of the torsion beam, scaled by the length of the cantilever beam).

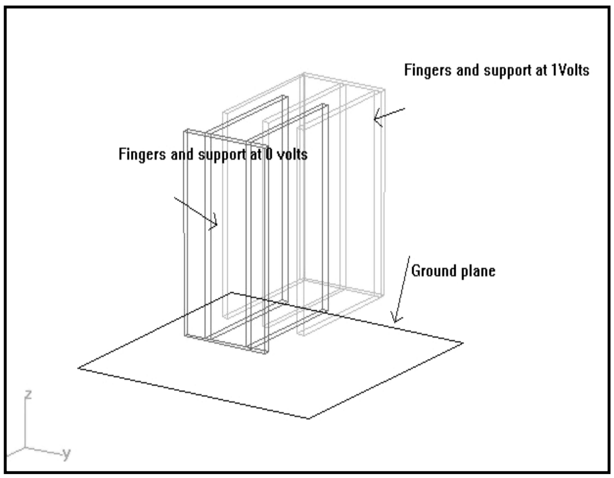

Figure 3 COULOMB model for electrostatic calculations: There are 3 movable comb fingers and 2 fixed fingers along with the ground plane. The forces in the center movable finger were used for calculations.

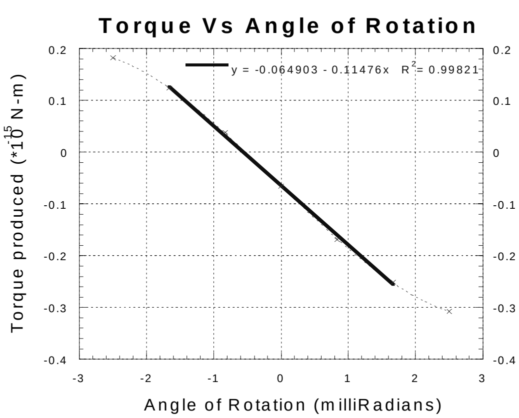

Figure 4 Torque produced as a function of the angle of rotation of the oscillator. Results from BEM simulations of model shown in Figure 3.

From the simulation results above, it should be noted that for the comb finger configuration in the torsional oscillator could be modeled, to the first order, as a linear 'electrostatic ' spring.

2.2 Mechanical Model

In order to estimate the values of the mechanical parameters of the system, namely the stiffness and Inertia, a Finite element model of the oscillator was used. The comb finger details were not included in this model. The material was assumed to be single crystal silicon. Two dimensional beam elements were used to model the oscillator and the anchor points were assumed fixed points.

The first few natural frequencies and mode shapes of the single oscillator was found using Finite element software (I-DEAS). The oscillators were designed such that the second and higher natural frequencies were far away from the first and not integral multiples of the first to avoid combination resonance effects in parametric excitation[Cartmell(1990)]. The moment of inertia was also calculated using a utility in the FE software. Figure 5 shows the first mode shape (corresponding to the lowest natural frequency) simulated. The second was an in plane 'twist' mode and the corresponding natural frequency was more than twice the natural frequency of the first.

By design, the second and higher mode frequencies were higher than the operating ranges of the oscillators and hence can be ignored in a first order model. Each of the oscillators was then modeled as a single degree of freedom mechanical oscillator (represented as a pendulum in Figure 6).

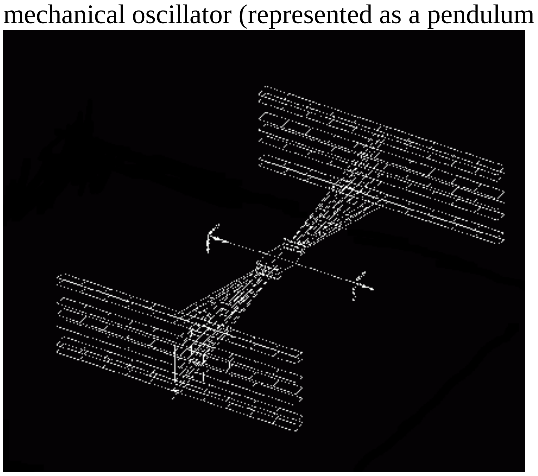

Figure 5: Frame capture of the displacements during the first natural mode of oscillation. In this mode, the displacements are purely perpendicular to the plane of the wafer. The oscillator motion can be compared to that of a "see-saw" about the axis of the supporting torsion beams.

2.3 System dynamics

Based on the simulations in the above two sections, a first order model is developed for the system. Figure 6 shows the schematic of the spring-mass equivalent system to the coupled MEM system. The springs here represent the electrostatic stiffness in the system from the interdigitated comb fingers and the pendula represent the MEM torsion beams.

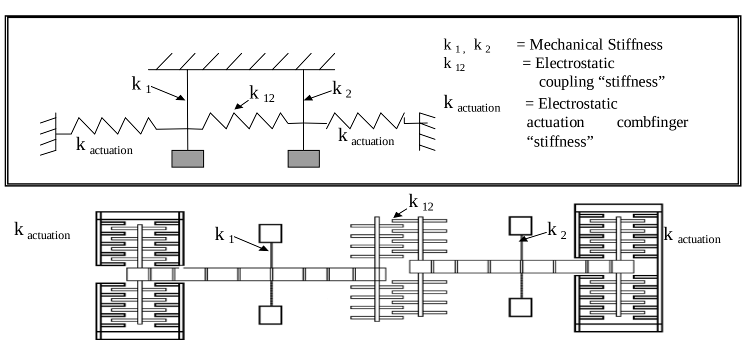

Figure 6: Schematic of the equivalent mechanical system and the MEM oscillator system.

The system as represented by Figure 6 has associated kinetic energy (***T***) and potential energy (***V***) as follows.

$$T = \tfrac{1}{2} I_1 \dot{\theta}_1^2 + \tfrac{1}{2} I_2 \dot{\theta}_2^2 \qquad \text{Eqn 1}$$

$$V = \tfrac{1}{2} k_1 \theta_1^2 + \tfrac{1}{2} k_2 \theta_2^2 + \tfrac{1}{2} k_{act1} (V_1 - V_2)^2 \theta_1^2 + \tfrac{1}{2} k_{act2} (V_3 - V_4)^2 \theta_2^2 + \tfrac{1}{2} k_{12} (V_2 - V_3)^2 (\theta_1 - \theta_2)^2 \qquad \text{Eqn 2}$$

$$F_1 = -c\dot{\theta}_1 \qquad \text{Eqn 3}$$

$$F_2 = -c\dot{\theta}_2 \qquad \text{Eqn 4}$$

where

$I_1, I_2 =$ mass moments of inertia of the oscillators,

$\dot{\theta}_1 = \frac{d}{dt}\theta_1$, $\dot{\theta}_2 = \frac{d}{dt}\theta_2$,

$\theta_1, \theta_2 =$ angular displacement of the oscillators,

$k_1, k_2 =$ mechanical stiffness of the oscillators,

$k_{act1}, k_{act2}, k_{12} =$ electrostatic stiffness of combfinger configurations,

V_1, V_2, V_3 and $V_4 =$ external potential applied at the contact pads

F_1 and F_2 are the damping forces proprtional to the velocities of either of the oscillators

c = viscous damping constant

In writing the energy terms, its been assumed that only the torsional degree of freedom is present and that the electrostatic stiffness varies linearly with rotation angle (as observed in the simulations) for small angle rotations. The square dependence of electrostatic force (and hence torque) on potential difference is a consequence of fundamental concepts in electrostatics and has been observed experimentally. For the most general case, the external applied voltage can be of any waveform - DC, harmonic wave, an impulse, triangular pulse etc. The damping in the system can be treated as external dissipative forces acting on the system. The mechanism of the damping is dependent on the ambient conditions and has been assumed viscous for the operating conditions of our test setup and verified by experimental observations.

Using Lagrange's equation [Greenwood (1988)] with the damping introduced as non-conservative forces, we can derive the following system of governing equations using equations 1-4.

$$I_1\ddot{\theta}_1+k_1\theta_1+k_{act1}(V_1-V_2)^2\left(\theta_1\right)+k_{12}(V_2-V_3)^2\left(\theta_1-\theta_2\right)+c\dot{\theta}_1=0 \qquad \text{Eqn 5}$$

$$I_2\ddot{\theta}_2+k_2\theta_2+k_{act2}(V_3-V_4)^2\left(\theta_2\right)+k_{12}(V_2-V_3)^2\left(\theta_2-\theta_1\right)+c\dot{\theta}_2=0 \qquad \text{Eqn 6}$$

This system of equations (5 and 6) governs the operation of the coupled torsional oscillator system assuming a single degree of freedom for each of the oscillators. These equations are coupled differential equations with time varying coefficients. Of interest is the case when the coupling strength is small, since the electrostatic stiffness is orders of magnitude smaller than the mechanical stiffness here.

Depending on the forms of external forcing voltages, the equations above will take the form of coupled forced harmonic equations with or without parametric forcing (a forcing function proportional to the angular displacement and periodic in time). Normal mode analysis can be done to understand the behavior of the case of non-parametric resonance and is available plenty in literature including [Meirovitch (1986)]. The response in a general case will be amplitude modulated harmonic functions or 'beats'.

Single torsional MEM oscillators have been extensively studied [Turner (1998)] and modeled. Of interest in some MEMS applications is the case when the oscillator motion is governed by Mathieu system [Nayfeh (1979), Cartmell (1990)]. Equations 5 and 6 reduce to a set of coupled Mathieu equations under certain external voltages. The case when the forcing functions are sinusoidal is of particular interest for resonant mode applications of MEMS. This system of equations will lead to simultaneous excitation of direct and parametric resonance for the case of sinusoidal input voltages as the electrostatic forces scale as the square of voltage difference. In order to isolate the parametric behavior of the system for easier experimental study, we use squarerooted sinusoidal input voltage signals.

In the following section, we'll discuss the analysis of the system governing equation for parametric forcing. The method of analysis is general and valid for many different forcing conditions. For demonstration, the algebraically simple case of coupled Mathieu system will be shown in this paper. Equations 5 and 6 reduce to the form 7 and 8 under such a case.

$$\textit{when} \quad V_1=V_2=0;\quad V_3=V_4=V_{ext}=\sqrt{A\cos(\Omega t)}$$

$$I_1\ddot{\theta}_1+ck_1\theta_1+k_{12}A\cos(\Omega t)\left(\theta_1-\theta_2\right)+c\dot{\theta}_1=0 \qquad \text{Eqn 7}$$

$$I_2\ddot{\theta}_2+k_2\theta_2+k_{12}A\cos(\Omega t)\left(\theta_2-\theta_1\right)+c\dot{\theta}_2=0 \qquad \text{Eqn 8}$$

For the case of no damping , scaling $\tau = \Omega t$,equations 7 and 8 can be rewritten as

$$\theta_1''+\varpi_1^2\theta_1+\varepsilon_1\cos\tau(\theta_1-\theta_2)=0 \qquad \text{Eqn 9}$$

$$\theta_2''+\varpi_2^2\theta_2+\varepsilon_2\cos\tau(\theta_2-\theta_1)=0 \qquad \text{Eqn 10}$$

where

$$\varpi_1^2=\frac{k_1}{I_1\Omega^2};\quad \varepsilon_1=\frac{Ak_{12}}{I_1\Omega^2}$$

$$\varpi_2^2=\frac{k_2}{I_2\Omega^2};\quad \varepsilon_2=\frac{Ak_{12}}{I_2\Omega^2}$$

Since the electrostatic 'stiffness' k_{12} is orders of magnitude smaller than the mechanical stiffness (obtained from the BEM and FEM simulations), the governing system of equations reduces to that of coupled second order differential equations with small time dependant (periodically forced) excitation – or Mathieu system of equations. Methods of perturbations will reveal qualitative features of the response and give directions for numerical solutions.

2.4 Analysis of the system equations: Method of two-variable expansion.

The method of two-variable expansion is a perturbation method where the solution of a weakly forced system is obtained by modifying the solution of an unforced system. In this method, the 'constant coefficients' of the harmonic solution of an unforced system is assumed to be varying in 'slow time'. This is equivalent to the method of multiple scales. The equations 7 and 8 can be rewritten as follows for the case of no damping. The perturbation analysis is done assuming no damping and the effect of presence of damping is discussed later.

For the case when the moment of Inertia of the two oscillators are same (but the mechanical stiffness is different), the small parameters ε_1 and ε_2 will be identical, say ε. The method of two-variable expansion involves starting with a solution for the system of equations 9 and 10 of the form

$$\theta_1(\tau,\eta) = a_1(\eta)\cos(\varpi_1\tau) + b_1(\eta)\sin(\varpi_1\tau) \qquad \text{Eqn} \quad 11$$

$$\theta_2(\tau,\eta) = a_2(\eta)\cos(\varpi_2\tau) + b_2(\eta)\sin(\varpi_2\tau) \qquad \text{Eqn} \quad 12$$

where

$$\eta = \varepsilon\tau\ , \quad a \quad slow \quad time - scale\ .$$

When these expressions are substituted in the governing equation, the condition for removal of the secular terms gives rise to the "slow-flow "equations - which are the equations governing the dynamics of the coefficients (a_1, b_1, a_2, b_2) in slow time η. These will be linear coupled equations and the time domain response will govern the amplitudes of motion of the oscillators. If the slow time response of the coefficients are sinusoidal, the overall response will be amplified and modulated, but will remain bounded. If the response blows up (exponential growth), then the amplitudes of the oscillator response will be exponentially growing sinusoids - which is unbounded response.

From the analysis of the above equations, it was found that, there are exponentially growing solutions for the following cases:

1.When the driving frequency is twice the natural frequency of either of the oscillator (uncoupled first order parametric resonance)

2.When the driving frequency is the sum of the two natural frequencies (coupled parametric resonance).

When the driving frequency is equal to the difference of the two resonance frequencies, the slow flow response is sinusoidal, thereby giving a frequency modulated output response.

It is known that parametric resonance (for a single oscillator case) has a dependence on amplitude of the drive, unlike direct resonance. For a single oscillator, it is known that with increasing amplitude of drive, there is exponentially growing response at a wider band of frequencies around the central parametric resonant frequency. In other words, there is a region of instability in the driving frequency, drive amplitude parameter space. The perturbation analysis is done at frequencies perturbed (by a small amount δ) off the instability curve ($\omega_1 + \omega_2 = 1$) perpendicular to the curve at each point, in order to map the instability zone in the non-dimensional parameters (ω_1^2,ω_2^2,ε) space. The perturbation analysis is performed on the following pair of equations subjected to the condition ω_1+ω_2=1.

$$\theta_1'' + \varpi_1^2 \theta_1 = -\varepsilon \ [\cos \tau(\theta_1 - \theta_2) + \frac{\delta \theta_1}{2\varpi_1}] \qquad \text{Eqn 13}$$

$$\theta_2'' + \varpi_2^2 \theta_2 = -\varepsilon \ [\cos \tau(\theta_2 - \theta_1) + \frac{\delta \theta_2}{2\varpi_2}] \qquad \text{Eqn 14}$$

This leads to the following instability zone in the $(\omega_1^2, \omega_2^2, \varepsilon)$ space (Figure 7)

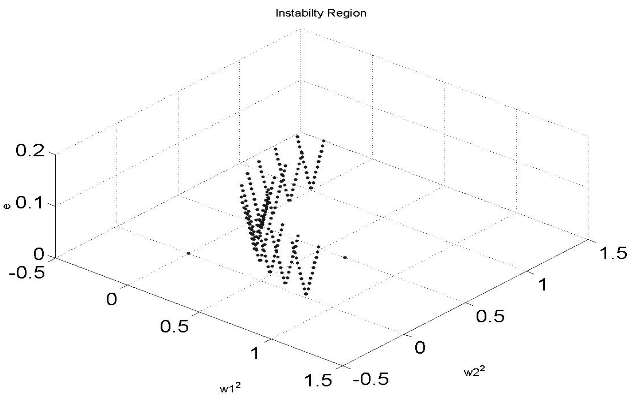

Figure 7: Representation of the instability boundary in the $(\omega_1^2, \omega_2^2, \varepsilon)$ parameter space. The region inside of the ' V' shape has parametric response while the outside region doesn't. This result is from first order perturbation analysis, which implies that it holds good only for small values of ε. (The Z-axis parameter)

The effect of damping, if small (i.e. same order of magnitude as the parametric forcing term) can be included in the two-variable expansion perturbation [Rand (2000)] .The effect of damping will be to introduce a minimum amplitude of forcing required for any parametric resonance to be observed. This will appear as the 'tongue' of instability curve looking like a 'U', rather than a 'V' as shown in Figure 8.

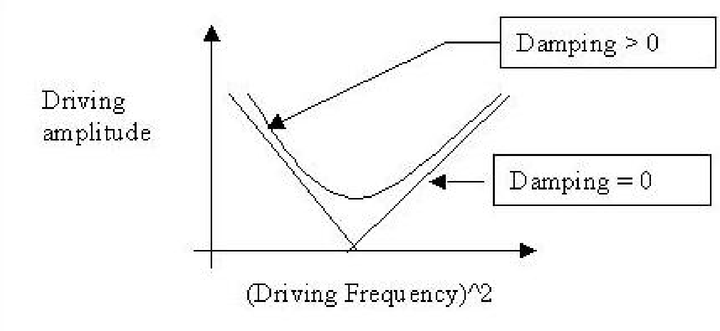

Figure 8: Schematic of the effect of damping on the instability curve as obtained from perturbation analysis.

The perturbation analysis for various cases of driving voltages was done using the symbolic Math software MACSYMA. The various cases of external forcing voltages lead to the same qualitative result of occurrence of coupled parametric resonance (exponentially growing sinusoidal solution) when driving at the sum of the natural frequencies and an amplitude modulated sinusoidal when driving at the difference of natural frequencies.

2.5 Numerical Analysis

The qualitative behavior of the oscillator system response is understood by perturbation method, and lends itself as a framework to do numerical simulations in the appropriate parameter space. The amplitude build up to resonance is slowed down, but nevertheless exponentially increasing for the case including damping. Figure 9 shows a comparison plot between the case of no damping and damping at parametric resonance.

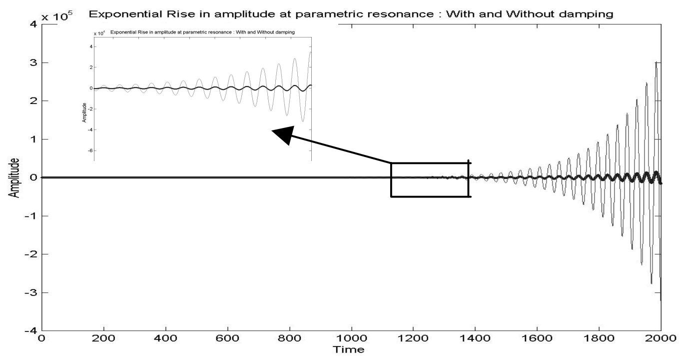

Figure 9: Numerical simulation result for the case of $w_1 + w_2 = 1$; [$w_1 = .3$, $w_2 = .7$]. The light trace is the case with no damping and the dark is the case with a non-dimensional damping factor of .2. This result is in close agreement of the analytical results for both the cases.

3. EXPERIMENTAL RESULTS

The experiments discussed here were performed using the high accuracy laser Doppler vibrometry technique[Turner (1999) b]. The devices were packaged in a 24-pin package and wirebonded to the contact pins. A voltage source (HP33120A) and a custom-made voltage amplifier were used to generate the excitation voltages. The displacement and velocity outputs from the vibrometer were recorded and analyzed with a HP Spectrum Analyzer (HP89470A) and Tektronics Oscilloscope (TDS 420 A). The schematic of the test set up is as shown in Figure 10. The Laser Doppler Vibrometer with built in controllers and sensor heads (Polytec, OFV-3001, and OFV- 511) uses a 633-nm wavelength, 205-mm cavity length Helium-Neon laser. The principle of measurement is optical interferometry. It is a heterodyne interferometer, meaning the velocity magnitude as well as the direction is determined by use of an additional Bragg cell[Polytec (1999)]. The controller in the sensor head demodulates the radio frequency signal from the sensor head and information about the velocity and displacement of the moving part focused is obtained from the frequency and phase respectively. This gives different limits on the velocity and displacement measurement ranges. In the experiments performed with the coupled oscillators, response measurements are in either displacement X (μm) or velocity V (mm/s) depending on the frequency of operation. It should be noted that for harmonic forced oscillations, either of the data gives the same information and is related by the frequency of response. [$X=A\cos(\omega t)$; $V=-A\omega\sin(\omega t)$]

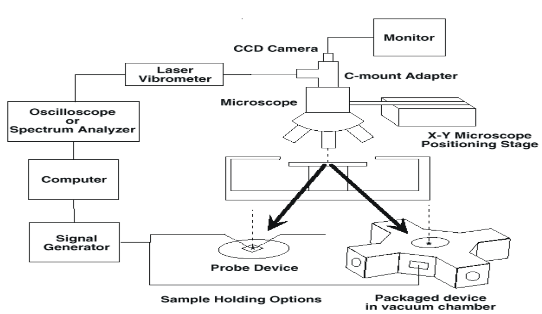

Figure 10: Schematic of the experimental set up used to measure real time displacement and velocity data of the moving oscillator.

The impulse response of the system [response to a burst signal of amplitude ~40V] was recorded real time. Amplitude modulated harmonic response (beats) and an exponential amplitude decay with time was observed (see figure 11). An input voltage of ~40V (Dark trace) was applied to one of the oscillators and its response (velocity data from the laser interferometer in mm/s) recorded real time. There are two characteristic frequencies (the harmonic and 'beat' frequency) associated with the response, which is typical of weakly coupled oscillator systems. It can also be seen that the system is underdamped, as there is an exponential reduction in amplitude of the oscillations.

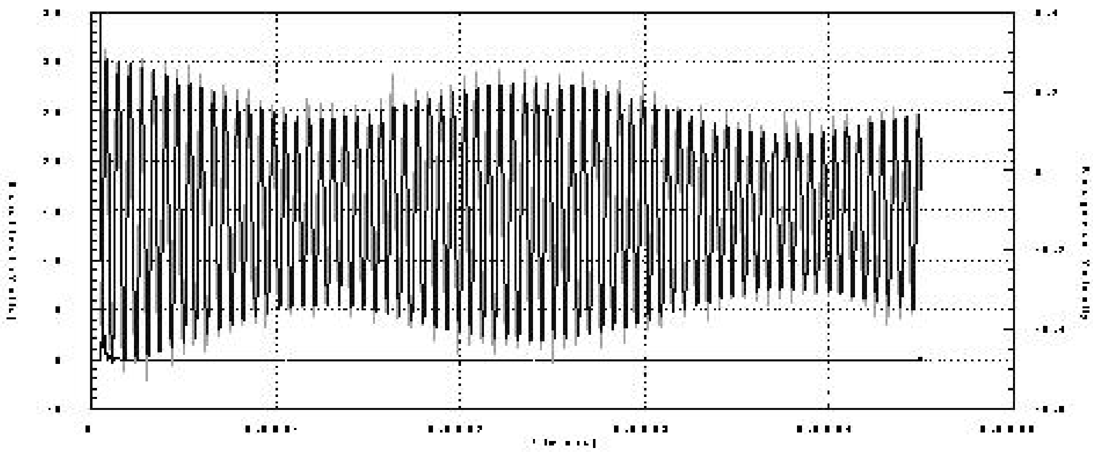

Figure 11: Response velocity data to impulse input and curve fitting (dark trace-least square fit <2%) of the response data with an equation of the form $A+Be^{2bwt}[\cos(wt+\varphi 1)*(\cos(\Omega t+\varphi 2)+\sin(\Omega t+\varphi 3))]$. The parameters that were extracted from the fit are the harmonic frequency component (*w*), beat frequency(Ω) and the exponential amplitude decay constant(b).

Parameter	**Fit from Impulse response (Experimental)**
Harmonic frequency (*w*)	154.48 **kHz**
Beat frequency (Ω)	24.76 **kHz**
Non-dimensional Damping co-efficient (b)	7.7736e-4

Table 2: Parameters extracted from the experimental data of Impulse response

The numbers from the above experiment verify the assumptions in the model that the electrostatic force and damping parameters are small by an order of magnitude compared to the mechanical parameter. Also, it illustrates one experimental procedure by which some of the system parameters can be extracted using the dynamic response of the system.

The behavior of the oscillator system response in the parametric instability "tongue" would be such that when driven close to the sum to the two natural frequencies, there will be response at two frequencies which add up to the driving frequency. This response was captured in one of the coupled oscillator systems in the experiments. The frequency domain response of the oscillator shown in Figure 12 captures the essence of such a response.

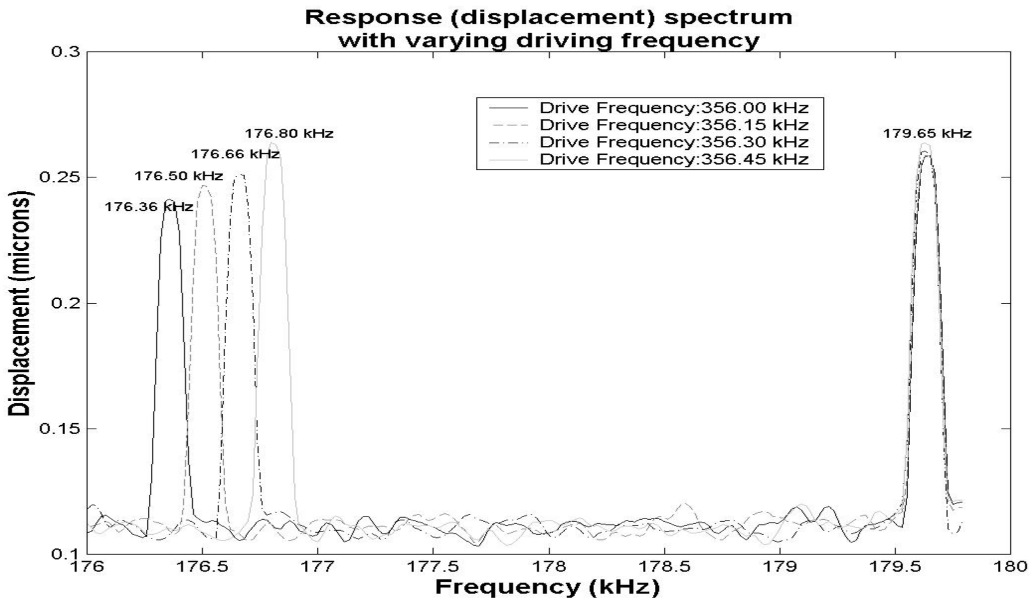

Figure 12: Response of the oscillator system to excitation with a square rooted sinusoidal forcing function with an amplitude of 20Vpk-pk. As the driving frequency is changed within the instability "tongue", the oscillator system response is such that the sum of the two response frequencies is equal to the driving frequency. The individual natural frequencies were independently found to be 176.5kHz and 179.65 kHz for the system tested here.

4. DISCUSSION

The analysis and experimental verification of the occurrence of coupled parametric resonance in this electrostatically coupled torsional MEM device has many implications for a range of resonant electrostatic MEM applications. Whenever there is a possibility of presence of a displacement-dependant force (parametric) generation, however small in magnitude, its effect can be significant. Asymmetric electric fields sometimes lead to parametric force generation. Some examples of such situations are parallel plate actuators, out-of-plane or 'levitation' comb drives, in-plane drive with non-interdigitated comb finger banks, buckled actuators/sensors (buckling may occur due to stress arising out of processing conditions). Obtaining a high degree of fabrication control in semiconductor processing is difficult and expensive.

Many dynamic responses that are not captured by a simple first-order model are often attributed to possible non-linearity in the system. The phenomenon of parametric resonance suggests that even in the absence of non-linearity, a rich dynamic behavior could result. The perturbation analysis predicts that parametric resonance will be excited only when ε (proportional to the electrostatic 'stiffness' k_{12}) is greater than the damping constant 'c'. This implies that when the damping is small ('Quality factor' is large), the possibility of occurrence of parametric effects is higher. One of the desirable qualities of MEMS based sensors is its high 'Quality factor', where even a small parasitic signal could trigger the parametric resonance.

There are many applications where the occurrence of coupled parametric resonance can be harnessed. One application arising from the coupled parametric response (described in figure 12) is a tunable frequency splitter. Parametric resonance plays a vital role in applications where both sensing and actuation are electrostatic. The fact that the drive and response frequencies are different in parametric resonance can be used to decouple the sense and actuation frequencies. A coupled system with natural frequencies that are incommensurate will have the drive and sense frequencies differentiated by any design amount.

CONCLUSION AND FUTURE WORK

Understanding the effects of electrostatic interactions is important for design of electrostatically actuated/sensed resonant mode MEMS. The model development and analysis of the dynamic characteristic of a coupled MEM system is presented. The modeling and analysis methods described here are useful tools in the design of many resonant mode MEMS. The perturbation analysis reveals the effect of small magnitude time varying stiffness introduced to the system through electrostatic interactions. Experimental evidence supporting the model is also presented. The system of coupled oscillators can be used as a test bench for understanding the micro-scale electrostatic transduction .

The idea of using dynamic characteristics like resonant frequencies and forced response to estimate system parameters (inertia, stiffness) is a classic experimental mechanics tool. Using parametric resonance behavior is an extension of this idea. The transition between stable and unstable region (Figure 7) is very sharp due to the exponential rate time dependence of the solution. Mapping the transition curve experimentally is presently under progress. The effect of varying different design parameters, like the distance between the comb fingers, the overlap length of the fingers, can be studied using such a configuration of coupled oscillators.

Bibliography:

Baskaran, R and Turner, K, L, **2000**, *Electrostatically coupled MEM oscillators*, Proceedings of the 2000 IMECE, MEMS –Vol2, p. 151-156.

Cartmell, M., **1990,** *Introduction to linear, parametric, and nonlinear vibrations*. 1st ed., London; New York: Chapman and Hall.

Eller, A.I., **1972**, *Fractional Harmonic frequency pairs in non-linear systems.* The journal of the Acoustical Society of America, **53**(3): p. 758-765.

Gabriel.K., J.J., Trimmer.W, **1990**, *Small machines*, *Large opportunities.* Micromechanics and MEMS Classical and Seminal papers to 1990(IEEE). .

Greenwood, D.T., **1988,** *Principle of dynamics*. Second Ed: Prentice-Hall Inc.

Hui, C.Y., J.L.A. Yeh, and N.C. Tien, **2000**, *Calculation of electrostatic forces and torques in MEMS using path-independent integrals.* Journal of Micromechanics and Microengineering, 10(3): p. 477-82.

Kovacs,G, 1998 , *Micromachined Transducers Sourcebook***,** WCB McGraw-Hill, ISBN 0-07-290722-3 .

Meirovitch, Leonard, **1986** "*Elements of Vibration Analysis*", pp. 125-129 McGraw-Hill –Mechanical Engineering Series

Miller, S.A., K.L. Turner, and N.C. MacDonald, **1997** , *Microelectromechanical scanning probe instruments for array architectures.* Review of Scientific Instruments, 68(11): p. 4155-62.

Nayfeh, A.H and. D.T.Mook., **1979** *Non-linear Oscillations*: John Wiley and Sons, NewYork.

Nguyen, C.T.C., **1999**, *Frequency-selective MEMS for miniaturized low-power communication devices.* IEEE Transactions on Microwave Theory and Techniques, **47**(8): p. 1486-503.

Polytec, **1999**, *Vibrometer User's manual*. .

Rand, 2000, *Lecture notes on non-linear vibrations*, Source: http://www.tam.cornell.edu/randdocs/nlvib34a.pdf

Saif, M.T.A. and N.C. MacDonald, 1**996**, *Micro mechanical single crystal silicon fracture studies torsion and bending*. Proceedings. IEEE, The Ninth Annual International Workshop on Micro Electro Mechanical Systems. An Investigation of Micro Structures, Sensors, Actuators, Machines and Systems (Cat. No.96CH35856)

Tang, W.C., Martin G. Lim, RogerT.Howe, **1992**, *Electrostatic Comb Drive Levitation and Control Method.* Journal of MicroElectro Mechanical System, **1**(4): p. 170-178.

Turner, K.L., *et al.*, **1998**, *Five parametric resonances in a microelectromechanical system.* Nature, **396**(6707): p. 149-52.

Turner, K., L., **1999 (a),** *Application and Analysis of Parametric Resonance in Micro Electro Mechanical Systems*, in *Theoretical and Applied Mechanics*. Cornell University: Ithaca.

Turner, K., L., P.G.Hartwell, R.Y.Webb, and N.C.MacDonald **1999 (b),** *Three-dimensional motion characterization using Laser Vibrometry*. in *Transducers '99- The 10th International conference on solid state sensors and Actuators, Japan, 7-10 June 1999*.