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NOISE ANALYSIS IN PARAMETRIC RESONANCE BASED MASS SENSING

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ABSTRACT

Mass sensing based on parametric resonance has shown high sensitivity and has many potential applications in chemical and biological sensing. We investigate noise effects on the sensitivity of mass sensing using parametric resonance. Temperature fluctuation noise, Johnson noise, and Brownian motion noise have been considered. Numerical simulation and experimental results show that noise affects the sensitivity of parametric resonance mass sensing in different mechanism from simple harmonic resonance based mass sensor and Brownian motion noise of the micro-oscillator has the major contribution to the frequency uncertainty at the boundary of parametric resonance area and the sensitivity in mass sensing.

INTRODUCTION

In resonance mode mass sensing applications, mass change causes the resonance frequency to shift. By tracking this resonance frequency shift information, mass change in the sensor system can be inferred. Due to the capability of achieving small mass and high resonance frequency, sensors with high sensitivity can be built using MEMS resonators, such as cantilevers and other electrostatic oscillators. In 2004, B. Ilic has succeeded detecting mass change in attogram level with a nano scale cantilever in high vacuum [1]. Many chemical and biosensors can be built based on mass sensing. In a micro-cantilever array, information on cantilever resonant frequency shifts can be used for recognition of a variety of chemical substances, including water, primary alcohols, and alkanes [2]. The mass of a single E. Coli cell (about 665 femtograms) has been detected on a 15 μm long, 5 μm wide and 320 nm thick cantilever with resonance frequency about 1.08 MHz [3].

The sensitivity of such sensors is decided by many factors, including total effective mass, resonance frequency, quality factor, and noise processes. Noise is an important factor on sensitivity in resonance mode mass sensing. In vacuum condition with high quality factor, noise processes cause the resonance frequency to fluctuate, and therefore, results in the uncertainty of the identification of the frequency. There are many noise sources that can contribute to this uncertainty. According to K.L. Ekinci [4], in the domain of micro/nano scale sensors, fundamental fluctuation processes, such as thermalmechanical fluctuation, temperature fluctuation, adsorption-desorption, and momentum exchange noise, are likely to dominant this uncertainly.

However, in a micro/nano oscillator working in high-pressure environment, such as in air or liquid environment, quality factor (Q), (due to damping from air, liquid, or even from thermal-elastic dissipation) is likely to be a dominant factor in the resolution of resonance frequency tracking. In high pressure sensing applications, such as in air or liquid, the resonance frequency shift resolution can be more than 1 orders magnitude lower than in vacuum condition because of low Quality factor.

Many methods have been proposed to improve the sensitivity of resonance mode mass sensors, such as using feed back control to improve quality factor [5] or building resonators with ultra high resonance frequency and very small mass working in low vacuum and low temperature environment. Parametric resonance amplification is another efficient technique in improving the performance of resonance mass sensor. In our previous work, we have demonstrated that mass sensing using micro-oscillator based on parametric resonance phenomenon has shown high sensitivity and has many potential applications in chemical and biological sensing [6-8]. In air pressure, the sensitivity can be improved 1~2 orders magnitude compared to the same sensor working in harmonic resonance mode. However, just as in many sensor applications, noise process is an important factor in determining the final sensitivity in such sensors.

In this work, we investigate noise effects on the sensitivity of mass sensing using parametric resonance phenomenon. Temperature fluctuation noise, Johnson noise, and Brownian motion noise have been considered. Numerical simulation and experimental results show that noise affects the sensitivity in different mechanism from simple harmonic resonance based mass sensor [9-11] and Brownian motion noise of the micro-oscillator makes the major contribution to the frequency uncertainty in a prototype parametric resonance based mass sensor.

PARAMETRIC RESONANCE MASS SENSING

In a mass-spring dynamic system with time-varying mass or stiffness term, parametric resonance can be actuated at certain frequencies [12]. In MEMS, examples include parallel plate capacitor type cantilevers driven by electrostatic force and non-interdigitated comb-finger driven oscillators [7,13]. Shown in Fig.1 is a prototype mass sensor to study parametric resonance mass sensing. The backbone of the oscillator is supported by four springs, which provide recovery force for the vibration. A pair of non-interdigitated comb-finger sets drives the backbone using electrostatic force, as a voltage signal is applied between fixed and moving comb-fingers. Because of the physical nature of non-interdigitated comb-fingers, electrostatic force generated not only depends on time, *t*, but also depends on the position of the backbone. Therefore, in vibration, the effective stiffness of the oscillator is tuned by applied AC voltage signal and is time-dependent. The first order parametric resonance can be excited as this stiffness tuning is twice the natural resonance frequency.

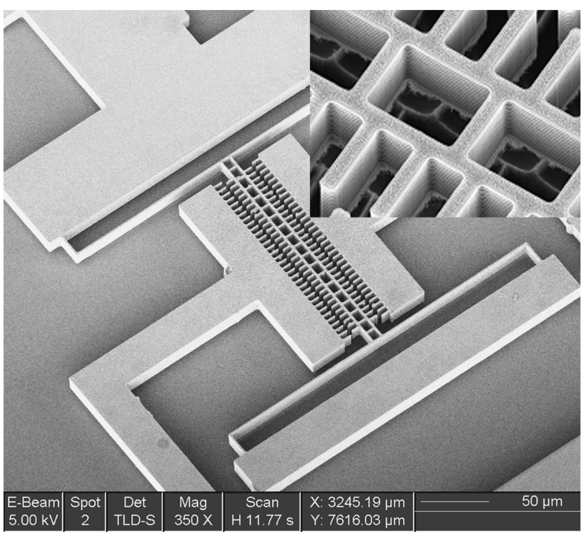

Figure 1 A prototype mass sensor. It is a micro-oscillator, driven by a pair of non-interdigitated comb-fingers.

As a square rooted AC voltage signal ($V_A(1+\cos 2\omega t)^{1/2}$) is applied across the non-interdigitated comb-fingers shown in Fig.1, the movement of the device is governed by nonlinear Mathieu equation [7]:

$$\frac{d^2x}{d\tau^2} + \alpha\frac{dx}{d\tau} + (\beta + 2\delta\cos 2\tau)x + (\delta_3 + \delta_3^{'}\cos 2\tau)x^3 = 0 \qquad (1)$$

where $\alpha = \frac{2c}{m\omega}$ $\beta = \frac{4(k_1 + r_1V_A^2)}{m\omega^2}$ $\delta = \frac{2r_1V_A^2}{m\omega^2}$

$$\delta_3 = \frac{4k_3 + 4r_3V_A^2}{m\omega^2} \qquad \delta_3^{'} = \frac{4r_3V_A^2}{m\omega^2}$$

m, k_1 and k_3 are the mass, linear and cubic mechanical stiffness of the oscillator, *c* is the damping coefficient, r_1 and r_3 are linear and cubic "electrostatic stiffness" and $\tau=\omega t$ is a normalized time [7].

Compared to harmonic resonance, parametric resonance has several unique characteristics [7,12]. First, it has well defined resonance area ("tongue"). Parametric resonance only happens inside the "tongue". Second, at one of the boundary of the "tongue", there is a sharp "jump" in frequency response. Third, damping changes the shape of the resonance area and raises the minimum power needed to actuate parametric resonance, but does not change the other characteristics. In other words, regardless of damping level, theoretically, parametric resonance can always be actuated.

These three characteristics make parametric resonance attractive in mass sensing application. Since the boundary of parametric resonance area, where the "jump" happens, depends on oscillator parameters, including stiffness, mass, and electrostatic force, any mass change in the oscillator causes the boundary to shift. By measuring this frequency shift, mass change can be found. Because of the sharp "jump", this frequency shift is very easy to measure and the resolution can be extremely high. In an earlier test, a resolution with *0.001 Hz* has been observed [12]. Furthermore, high sensitivity can be achieved not only in vacuum, but also in high damping environment such as in air, which makes parametric resonance more efficient than harmonic resonance based mass sensing.

The frequency at the boundary of parametric resonance area, where the "jump" happens, is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{4k_1 + 2r_1V_A^2}{m}} \qquad (2)$$

and the relationship between mass change (Δm) and resonance frequency shift (Δf) is

$$|\Delta m| - 2m\frac{|\Delta f|}{f_0} \qquad (3)$$

NOISE CHARACTERIZATION

As with most micro-sensors [10,11,14], noise is an important issue in mass sensing using parametric resonance technology as well. The ability to detect ultra-fine frequency shift, and therefore the mass sensitivity, is affected by noise processes in the oscillator. We use laser vibrometry [15] to measure the frequency response of parametric resonance and look for the frequency when the "jump" happens. The measurements are taken in fixed time intervals and certain amounts of such measurements are recorded and analyzed statistically.

Figure 2 shows a measurement of frequency shift at the boundary of parametric resonance in about half an hour. Along with the average frequency shift, there is certain distribution of the measurement around this average value In our earlier test, less than *0.001 Hz* of frequency shift size can be detected using parametric resonance [16]. However, the frequency uncertainty (fluctuation), as shown in Fig.2, can be much larger than *0.001 Hz*.

Figure 3(a) shows two measurement of this frequency fluctuation of the device shown in Fig.1 with driving voltage at *16 V* using square root sinusoidal signal ($V = V_A\left(1+\cos 2\omega t\right)^{1/2}$). The standard deviation of the frequency fluctuations is about *0.8 Hz* at room temperature and air pressure.

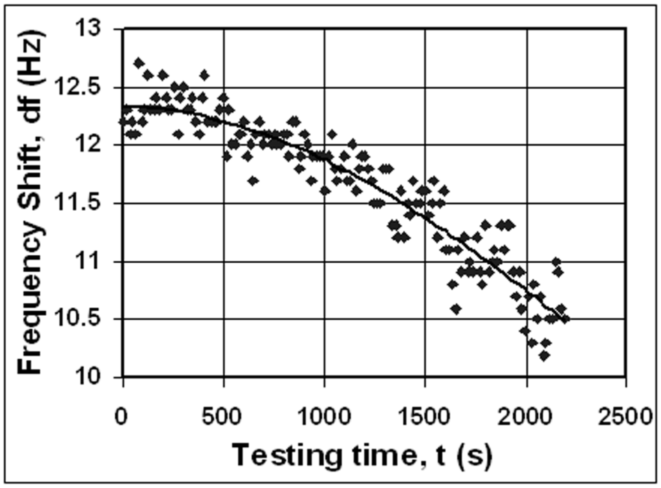

Figure 2 A measurement of frequency shift at the boundary of parametric resonance. There is certain distribution of the measurement around the average frequency shift value.

ANALYSIS AND SIMULATION

Many factors can contribute to this frequency fluctuation, which will ultimately affects the sensitivity of mass sensing. Different from traditional methods of measuring the resonance frequency in harmonic resonance mass sensing using phase locked loop (PLL) or positive feedback method [4], we use optical method to monitor the frequency shift at the boundary of parametric resonance area by manually recording the driving frequency when the 'jump' in frequency response happens. Therefore, noises causing the resonance frequency to fluctuate because of frequency or phase modulation, such as thermomechanical fluctuation noise, will not be considered to significantly affect the accuracy of the frequency measurement.

From Equation (2) and (3), the frequency at the boundary of parametric resonance tongue depends on mass m, stiffness k, and driving voltage V_A. Any noise source causing these parameters to fluctuate can affect the accuracy of mass sensing. Since Young's modules E is temperature-dependent [11,17], temperature fluctuation causes the stiffness to fluctuate and can be a noise source in parametric resonance mass sensing. Johnson noise in the driving circuit, in which the oscillator actually works as a capacitor, adds a voltage fluctuation to the driving voltage signal. Because the oscillator is driven by a pair of non-interdigitated comb fingers, the effective stiffness of the oscillator can be tuned by the driving electrical signal variation [18]. Therefore, Johnson noise can as well cause the frequency fluctuation at boundary of parametric resonance area as shown in Equation (2).

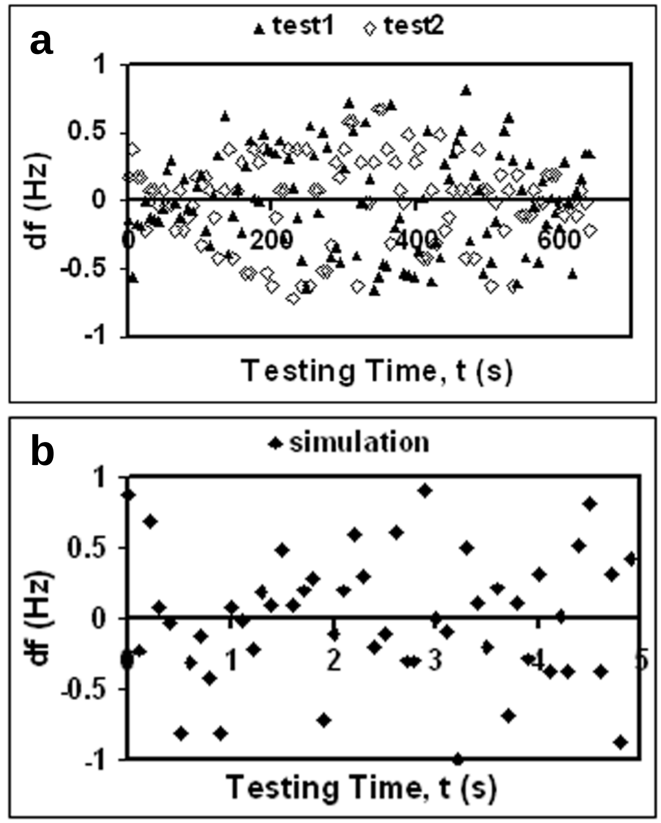

Figure 3 Frequency fluctuations at the boundary of parametric resonance in (a) two experimental tests and (b) numerical simulation caused by Brownian motion noise.

Brownian motion noise can be another noise source in parametric resonance mass sensing. This random motion results from the molecular agitation in air surrounding the oscillator. The 'jump' event not only depends on the system parameters, but also depends on the motion level before the driving frequency approaches the boundary. From dynamics point of view, this random movement actually works as initial value, which will decide whether the 'jump' will happen at next driving frequency in the sweep. According to the dynamics of parametric resonance (shown in Fig.4) [7], area III is a bi-stable area with one trivial solution and one non-trivial solution. As mentioned earlier, we manually record the frequency shift information using optical method. Due to the random Brownian noise, we have never been able to measure the exact boundary frequency. The 'jump' always happens at slightly higher frequency and this frequency fluctuates due to the random motion of the oscillator.

The temperature fluctuation noise effect on frequency fluctuation has been discussed by Vig *at el* [11], Cleland *at el* [9], and Ekinci *at el* [4]. The oscillator is made of silicon and has resonance frequency *50 kHz*, mass *30 ng*, total volume *20,000* μm^3, and working temperature *300 K*. The estimated frequency fluctuation standard deviation caused by temperature fluctuation noise is less than *0.1 Hz*, which is not significant compared to the testing result, shown in Fig.3.

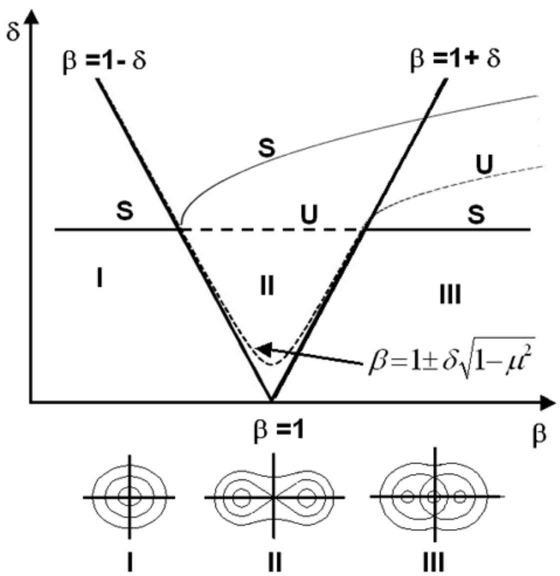

Figure 4 Dynamics of parametric resonance. The β-δ plane can be divided into three area, I, II, and III. According to phase plane in area III, the stable value can be trivial and non-trivial, which depends on initial value. Initial value variation can cause uncertainty of the right boundary of area II.

So far, no analytical method has been found to estimate Johnson noise and Brownian motion noise effects on parametric resonance mass sensing. The motion of the oscillator is governed by nonlinear Mathieu equation (Eq. (1)). It is a nonlinear second order differential equation. We use numerical method to study the frequency fluctuation caused by Johnson noise and Brownian motion. The extra voltage fluctuation caused by Johnson noise is treated as a white noise and added to the driving voltage signal in the simulation. The random motion of the oscillator, which results from Brownian motion noise, is treated as a white noise as well. This white noise is input as initial value in Matlab ODE solver.

According to Equipartition Theorem [10], the extra voltage fluctuation caused by Johnson noise can be estimated using following equation:

$$\frac{1}{2}C\left\langle V^2\right\rangle=\frac{1}{2}k_BT \quad (4)$$

here, C is the capacitance between fixed comb-fingers and moving comb-fingers of the oscillator, V is the extra voltage fluctuation, Boltzmann constant k_B*=1.38e-23 W.sec/K*, and T is the working temperature. The capacitance C of the oscillator shown in Fig.1 is estimated to be *1 fF* (*1e-15 F*), therefore V_{rms} *= 2 mV* at room temperature (*300 K*). Using numerical method, the standard deviation of frequency fluctuation at driving voltage *16 V* is less than *0.05 Hz*.

Brownian noise can be estimated using Equipartition Theorem as well [10]. For an oscillator with stiffness *k*, the random motion *x* caused by Brownian motion noise is:

$$\frac{1}{2}k\left\langle x^2\right\rangle=\frac{1}{2}k_BT \quad (5)$$

The stiffness of the oscillator *k* is about *3 N/m*. The estimated x_{rms} equals *0.4 Å* at room temperature (*300 K*). Figure 3(b) shows the numerical simulation results of frequency fluctuation at the boundary of parametric resonance region caused by Brownian motion noise with x_{rms} about *0.4 Å*. The standard deviation of the frequency fluctuation is about *0.7 Hz*.

DISCUSSION

In parametric resonance mass sensing, temperature fluctuation noise, Johnson noise, and Brownian motion noise, can cause the frequency uncertainty at the boundary of parametric resonance area in the measurement. According to analytical and numerical calculation, at room temperature, the frequency fluctuation caused by temperature fluctuation noise and Johnson noise is insignificant compared to the experimental results, while that caused by Brownian motion noise is in the same level as the experimental results. Therefore, in parametric resonance mass sensing using the oscillator shown in Fig.1, Brownian motion noise is the major noise source.

In parametric resonance mass sensing, one of the advantage over harmonic resonance mass sensing is that damping effects on the sensitivity is insignificant theoretically from dynamic point of view [7]. However damping effect on frequency fluctuation is of great concern as well since it dramatically affects the mass sensitivity of harmonic resonance mode mass sensor. Here we experimentally measure the frequency fluctuation level at the boundary of parametric resonance area at different pressure, as shown in Fig.5. As pressure changes from *10 mTorr* to air pressure, the standard deviation of frequency fluctuation shows no sign of pressure dependence. This is understandable, since the major noise source in this case is Brownian motion noise, which is only temperature-dependent. Therefore, unlike harmonic resonance based mass sensor, it is possible to achieve the same mass sensing level in air pressure as in vacuum using parametric resonance mass sensing.

Certainly, other factors can also contribute to parametric resonance mass sensing, especially environmental factors, such as humidity fluctuation and temperature drift. In certain cases, they can be dominant, when the test lasts hours or days. However, these factors are extrinsic and not fundamental to the sensor itself. By using oscillator array and optimizing the design of sensors and reference sensors, these noise effects can be minimized when the measurement is taken in short time.

CONCLUSION

Noise processes, including temperature fluctuation noise, Johnson noise, and Brownian motion noise, in parametric resonance mass sensing is investigated both numerically and experimentally in this work. Brownian motion noise is the major contribution in the frequency fluctuation at the boundary of parametric resonance area and mass sensing in the proposed prototype mass sensor working in parametric resonance mode. Damping has insignificant effects on the noise floor. Therefore, unlike harmonic resonance mass sensing, testing environmental pressure does not dramatically affect the sensitivity of parametric resonance mass sensing. Same sensitivity level can be achieved in air pressure environment as in vacuum condition based on parametric resonance.

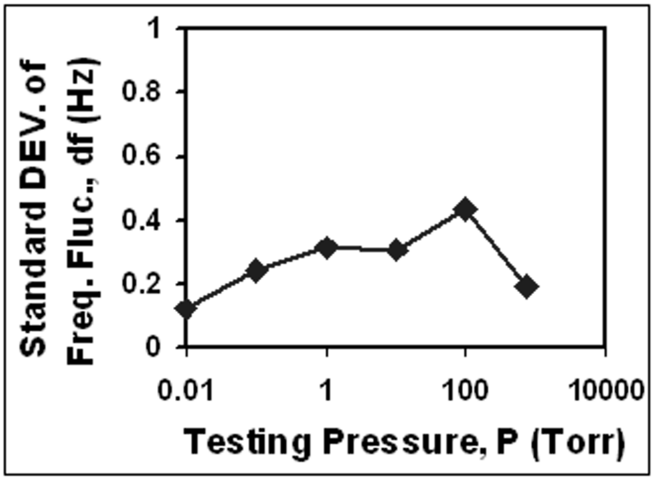

Figure 5 Frequency fluctuations at the boundary of parametric resonance at different pressure.

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