Problem 1. Consider a ¼-car suspension model with the following parameters:
M = 250kg
M = 15kg
Kt = 100,000 N/m

a) Derive Z(s)/Zr(s) (Hint: replace d/dt with s in the differential equations and assume initial conditions are zero. Replace s with jω to get equation 5-14 as a check)

\[
\begin{align*}
\text{Equation 5-12: } & M \ddot{z} + C_s \dot{z} + K_s z = C_s \dot{u} + K_s z u + F_d \\
& \left( M s^2 + C_s s + K_s \right) Z(s) = \left( C_s s + K_s \right) Zu(s) \tag{Eq. i} \\
\text{Equation 5-13: } & m \ddot{z} u + C_s \dot{z} u + (K_s + K_k) z u = C_s \dot{z} + K_s z + K_0 z r + F_d \\
& \left( m s^2 + C_s s + (K_s + K_k) \right) Zu(s) = \left( C_s s + K_s \right) Z(s) + K_0 Z r(s) \tag{Eq. ii} \\
\end{align*}
\]

Next, rearranging Eq. i:
\[
Zu(s) = \frac{\left( M s^2 + C_s s + K_s \right)}{\left( C_s s + K_s \right)} Z(s)
\]

Combining with Eq. ii:
\[
Z(s) = \frac{\left( m s^2 + C_s s + (K_s + K_k) \right) \left( M s^2 + C_s s + K_s \right) - \left( C_s s + K_s \right)^2}{\left( C_s s + K_s \right)} Zr(s) = K_0 Z r(s)
\]
\[ \frac{Z(s)}{Z_r(s)} = \frac{k_i (c_s + k_i)}{(m^2 + c_s + (k_i + k_2))(m^2 + c_s + k_i) - (c_s + k_i)^2} \cdot \left[ \frac{1}{m^2} \right] \]

\[ = \frac{k_i (c_s + k_i)}{(k_s^2 + c_s + k_i + k_2)(c_s^2 + c_s + k_2) - (c_s + k_i)^2} \]

* where \( k, c, k_i, k_j \) are defined on page 151 in text.

\[ \frac{Z(s)}{Z_r(s)} = \frac{k_i k_2 + k_i c_s}{k_s^4 + (1 + X) c_s^3 + (k_2 X + k_2 - c_s + k_i + k_2) c_s^2 + (k_i c + k_2 c + k_2 (c - 2 k_i c)) c_s + (k_i k_2 + k_2^2 - k_2^2)} \]

Therefore:

\[ \frac{Z(s)}{Z_r(s)} = \frac{k_i k_2 + k_i c_s}{k_s^4 + (1 + X) c_s^3 + (k_2 X + k_1 + k_2) c_s^2 + k_i c_s + k_i k_2} \]

Check for consistency with 5-14:

\[ s = jw \]

\[ \frac{Z(s)}{Z_r(s)} = \frac{k_i k_2 + j[k_i c w]}{k_s^4 - (k_i + k_2 X + k_2) c_s^3 + k_i k_2} + j[k_i c w - (1 + X) c_s c_w^3] \]

* checks to equation 5-14
b) Design Cs and Ks to for a performance car - explain. Plot the Bode plot from 0.01 to 100 Hz. (you may need to iterate your design).

Neglecting \( k_2 \)...

\[ \omega_n = \sqrt{\frac{k_s}{M}} = 2\pi \times 2 \text{ Hz} \]

\[ \frac{k_s}{M} = (4\pi)^2 \]

\[ k_s = (4\pi)^2 / M \]

\[ k_2 = \frac{k_s}{M} = (4\pi)^2 \approx 158 \]

\[ S = \frac{C_\text{s}}{2\sqrt{k_s M}} = \frac{1}{2} \]

\[ = \frac{C}{2\sqrt{k_2 M}} = \frac{C}{2\sqrt{158}} = \frac{1}{2} \]

\[ \therefore \quad \frac{C^2}{k_2} = 1 \]

\[ \Rightarrow \quad C = \sqrt{k_2} \]

\[ = \sqrt{158} \]

\[ C = \frac{4\pi}{2} \]

\[ \chi = \frac{\omega}{\omega} = 0.06 \]

\[ k_1 = \frac{k_s}{M} = \frac{100,000}{250} = 400 \]
Using Bode in Matlab.
Problem 2. Using the suspension model of problem one, consider the effect of an imbalance mass on the acceleration of the mass M. Use the following parameters:

Imbalance mass = 0.1kg at a radius of 0.2m  
Wheel radius = 0.2m  
Tire radius = 0.35m

a) find the Laplace transfer function from wheel force to acceleration of the mass M in symbolic form. (you can also replace \( j\omega \) with s and \(-\omega^2\) with \(s^2\) etc in equation 5-15 and divide through by \(M\))

b) plot the amplitude of the imbalance force as a function of vehicle speed, \(v\), for \(v = 0\) to 50 m/s.

c) plot the acceleration amplitude of the mass M as a function of vehicle speed

\[
\begin{align*}
\frac{\ddot{z}}{F_w} &= \frac{-k_1s^2 - Cs^3}{M\left[K_s^4 + (1+\lambda)Cs^3 + (k_2 + k_1 + k_2) s^2 + k_1Cs + k_1k_2\right]} \\
\text{Cons (a)}
\end{align*}
\]

\[
\begin{align*}
F = m \cdot \frac{v^2}{r} = \frac{0.1 \cdot v^2}{0.2} = \frac{v^2}{2}
\end{align*}
\]
C) Acceleration Amplitude =

\[ \left| \frac{\hat{Z}(j\omega)}{F_w(j\omega)} \right| \cdot \frac{\omega^2}{m_{imbal.} \cdot \frac{W^2}{V_{wheel}}} \]

\[ \text{where} \quad \omega = \frac{V}{r_{tire}} \]

```
k=158; R
r_tire=0.35;
r_wheel=0.2;
m_imbal=0.1;
numer=[c -kk 0 0];
denom=M*[x (1+chi) (kk*chi+k+kk) k*chi k*kk];
for vel=1:50
    w=vel/r_tire;
    F(vel)=m_imbal*w^2*r_wheel;
    a(vel)=abs(polyval(numer,li*w)/polyval(denom,li*w))*F(vel);
    v(vel)=vel;
end
plot(v,F);
figure
plot(v,a);
```