Problem 1. Show that lb·sec²/in has units of mass.

\[ F = ma \]
\[ F \times \left[ \text{lbs} \cdot \frac{\text{in}}{\text{sec}^2} \right] \]
\[ m \times \left[ \text{lbs} \right] \]
\[ a \times \left[ \frac{\text{in}}{\text{sec}^2} \right] \]

And
\[ m = \frac{F}{a} \times \left[ \frac{\text{lbs} \cdot \text{in}}{\text{sec}^2} \right] \]

\[ m \times \left[ \frac{\text{lbs} \cdot \text{sec}^2}{\text{in}} \right] \]
Problem 2. In class, we derived the $F_{x,\text{max}}$ equation, the maximum acceleration traction force (equation 2-23 in the text). Fill in the step that steps that I skipped in deriving the equation.

\[
F_{x,\text{max}} = 2 \mu \frac{W_r}{L} = 2 \mu \left( \frac{W_b}{2L} + \frac{F_{x,h}}{2L} - \frac{F_{x,v}}{Nf} \frac{r}{k} \right)
\]

\[
F_{x,\text{max}} \left( \frac{2\mu v}{Nf} \frac{r}{Ko} - \frac{\mu h}{L} + 1 \right) = M \left( \frac{W_b}{L} \right)
\]

\[
F_{x,\text{max}} = \frac{M \left( \frac{W_b}{L} \right)}{\left( \frac{2\mu v}{Nf} \frac{r}{Ko} - \frac{\mu h}{L} + 1 \right)}
\]

ans (2) √
Problem 3. Go to the howstuffworks.com website and look up differentials and limited-slip mechanisms. Describe in a sentence or two each mechanism.

Clutch-Type Limited Slip Differential

This differential has all of the same components as an open differential, but it adds a spring pack and a set of clutches. Differentiation between the wheels is a functions of spring force and clutch friction.

Viscous Coupling

This differential is usually found in four wheel drive vehicles. Two sets of plates are used with a viscous fluid to transmit torque.

Locking Differential

Generally used in off-road vehicles, a locking differential uses some electrical, hydraulic or pneumatic mechanism to lock the output pinions together.

Torsen Differential

The Torque Sensing differential mechanically senses torque between the two output shafts. A bias ratio is set which automatically gives more torque to the wheel with more traction.
Problem 4. Let's add a transmission to problem 4 from last week: a 1000kg vehicle, in 2\textsuperscript{nd} gear, can apply a maximum traction force of \( F(v) = 10,000 \left[ 1 - \left( \frac{v - 15}{30} \right)^2 \right] \) Newtons.

The vehicle drive-train has the following properties:
Nf = 3.5
Nt = 3.0, 2.0, 1.5, 1 for 1\textsuperscript{st}, 2\textsuperscript{nd}, 3\textsuperscript{rd}, and 4\textsuperscript{th} gear respectively

a) Using Matlab, plot the engine torque curve (N-m versus rad/sec).
b) Plot the Fx (N-m) curves verses velocity (m/sec) on a single graph for the four gears.
c) Assume zero drive-train inertia; find the optimum shift points in rads/sec and RPM.
d) What is the minimum 0 to 60 m/sec time with zero drive-train inertia?
e) Assume that the engine inertia is 2kg-m\(^2\) and that there is zero inertia for the remaining rotating components. What is Mr (kg) for the 4 gear ratios?
f) What are the optimum shift points when Mr is taken into consideration?
g) What is the minimum 0 to 60 m/sec time when the engine inertia is modeled? (When shifting up, assume that the engine drops speed instantaneously without a torque impulse to the drive-train.)

\[
\text{Force} = \text{Torque} \\
F = \frac{TeN_eN_f}{r} \\
\text{Velocity} = \text{Engine Speed} \\
\omega = \frac{vN_eN_f}{r}
\]
a) Using Matlab, plot the engine torque curve (N-m versus rad/sec)

1. Write a function to use force to calculate torque in second gear.

   function [torque] = Te(omega)
   NF = 3.5;  % differential gear ratio
   Nt2 = 2.0;  % gear ratio for second gear
   r = 0.3;  % tire radius
   v = r*omega/(Nt2*NF);
   torque = (r/(Nt2*NF))*10000*(1-{(v-15)/30}^2);
   end

2. To plot the graph:

   clear
   hold off
   % plot the engine torque curve from 0 to 9700 RPM
   for n=1:97
       rpm = 100*(n-1);
       omega=rpm*2*pi/60;
       y(n) = Te(omega);
       x(n) = omega;
   end

   plot(x,y)
   xlabel('Engine Speed (rad/sec)')
   ylabel('Torque (N/m)')
   title('Problem 4a - Engine Torque Curve')
ANSWER (a):
b.) Plot \( F_x \) (N) curves versus velocity (m/sec) on a single graph for the four gears.

```matlab
figure %create a new figure for shift curves

Nt = [3 2 1.5 1]; %vector of gear ratios
Nf = 3.5;
r = 0.3;
shift = [0 0 0]; %vector to keep track of when shifts occur

% create matrix of traction forces for various gears
% keep track of time that a higher gear provides more force

for n=1:10000
    v=n/100;
    for i = 1:4
        omega(i) = Nt(i)*Nf*v/r;
        F(i,n) = Te(omega(i))*Nt(i)*Nf/r;
        x(n) = v;
    end
    for i = 1:3
        if shift(i) == 0 %if the ith shift hasn't occurred
            if F(i,n)< F(i+1,n) %check to see if the next gear is better
                shift(i)=1; %if so indicate that the ith shift has occurred
            end
            shiftpoint{i}=omega(i); %save the shift point
            shiftpointv{i}=v; %save the shift point
        end
    end
end
hold on
axis([0 100 0 16000])
plot(x,F)
xlabel('Speed (m/s)')
ylabel('Force (N)')
title('Problem 4b - Force Curves')
shiftpoint % print the shift points in rad/sec
shiftpoint/2/pi*60 %print the shift points in RPM
shiftpointv % print the corresponding vehicle speed (m/sec)
hold off
```
ANSWER (b):

\[ \text{Speed (m/s)} \]

\[ \text{Force (N)} \]

c.) Assume zero drive-train inertia; find the optimum shift points in rad/sec and RPM.

Shift points are intersections of force plots:

<table>
<thead>
<tr>
<th>Shift</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1\textsuperscript{st} Shift</td>
<td>777.0 rad/sec = 7420 RPM</td>
</tr>
<tr>
<td>2\textsuperscript{nd} Shift</td>
<td>743.6 rad/sec = 7101 RPM</td>
</tr>
<tr>
<td>3\textsuperscript{rd} Shift</td>
<td>776.8 rad/sec = 7418 RPM</td>
</tr>
</tbody>
</table>
d.) What is the minimum 0 to 60 m/sec time with zero drive-train inertia?

**ANSWER (d)**

0-60 m/sec time = 8.65 sec.
e.) Assume that the engine inertia is 2 kg·m² and that there is zero inertia for the remaining components. What is $M_r$ (kg) for the 4 gear ratios?

**ANSWER (e):**

Combining equations 2.9 and 2.11a we see that $M_r = [(2*N_l^2*N_t^2/\tau)]^2$

| $M_{r1}$ | 2450 kg |
| $M_{r2}$ | 1088 kg |
| $M_{r3}$ | 612 kg  |
| $M_{r4}$ | 272 kg  |
f.) What are the optimum shift points when $M_r$ is taken into consideration?

The Force/$M_r$ (acceleration) graph is shown below:

From this plot it is obvious that 1\textsuperscript{st} gear should be skipped.

**ANSWER (f):**

- Gear 2 to Gear 3: 4470 RPM
- Gear 3 to Gear 4: 6051 RPM
g.) What is the minimum 0 to 60 m/sec time when the engine inertia is modeled? (When shifting up, assume that the engine drops speed instantaneously without a torque impulse to the drivetrain.)

To find the minimum time to 60 m/sec, the maximum acceleration needs to be selected. This is start in second gear and make one shift to third. By doing this and integrating the acceleration curve (as we did in homework #1) the velocity versus time curve is shown below:

![Graph showing velocity versus time]

**ANSWER (g):**

\[\text{Time}(0 \text{ to } 60 \text{ m/sec}) = 15 \text{ seconds.}\]